MONTE CARLO SIMULATION FOR RELIABILITY ANALYSIS OF COMPLEX SYSTEMS

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Abstract: Nowadays, most of the systems are very complex (complex structure, various probability distributions of individual components, etc.) and it is quite difficult to ensure their reliability. This paper deals with modern approach of quantitative reliability analysis – the Monte Carlo simulation. This approach is used to evaluation of the reliability parameters of the system. The knowledge of these parameters is suitable for the prevention of the frequently failures which means a higher lifetime and cost savings for producer.

Keywords: reliability, Monte Carlo, reliability analysis, MTBF

1. INTRODUCTION

Reliability is defined as the capability of a system to perform a required function. This capability (or attribute) constitutes a complex property of the given system, and it is expressed via reliability indicators. The most widely used reliability indicators are probability of failure $Q(t)$, failure probability density $f(t)$, probability of reliability $R(t)$, failure rate $\lambda(t)$ and mean time between failures *MTBF*. [4]

The overall calculation of these indicators is carried out using quantitative reliability analyses, which are based on creating a reliability model and applying certain rules to calculate the indicators. These analyses are suitable tools to express total reliability (the value of an indicator), lifetime, and other aspects.

The Monte Carlo simulation was developed in the 1940s by scientists at the Los Alamos National Laboratory for modelling of the random diffusion of neutrons. The name refers to the Monte Carlo Casino in Monaco. Today, the Monte Carlo is used in a wide array of application. One of these applications is just the quantitative reliability analysis. This technique differs from the other approaches in that it is not an analytical method but a statistical computer simulation tool. [1], [6]

2. THE MONTE CARLO RELIABILITY ANALYSIS

Monte Carlo is a quantitative reliability analysis. The result of this analysis is one of the reliability indicators (probability of failure $Q(t)$, failure rate λ, mean time between failures *MTBF*, and so on). For calculation these reliability indicators must be created mathematical model of analyzed system. Create this model is based on knowledge of reliability model (it is possible to use the Reliability block diagram), failure rate λ_i (or the other reliability indicator) of individual components and distribution of probability. Monte Carlo approach is characteristic with its versatility in terms of distribution of probability. It is possible to apply Monte Carlo on the system with various probability distributions of its elements. It is also its main advantage. [1], [2], [6]

The principle of this method is a statistical simulation of the behaviour of the modelled system. For this simulation is needed sufficiently large set of random numbers and is important to have a good random number generator. It is possible to use e.g. the MATLAB random number generator, based on the algorithm - Marsaglia's Random Number Generator. The random numbers from interval (0,1) generated with a random number generator are distributed with a normal distribution of probability. These numbers y_i must be transformed to numbers x_i with a required distribution and with probability density $f(x)$ by (1).

$$y_i = \int_{-\infty}^{x_i} f(x)dx \qquad [-] \qquad (1)$$

Random numbers with a required distribution x_i are thus given by solution of equation (1). Then are realized many experiments with a mathematical model of the analyzed system. This mathematical model is function $Z(x_i)$ of random variables x_i with a distribution of probability $F(z)$. The result of n experiments is set of random numbers z_i with a distribution of probability $F_n(z)$. For sufficiently large count of experiments, where count of experiments $n \rightarrow \infty$, is distribution $F_n(z)$ very nearing to distribution $F(z)$. The all process (algorithm) is shown on the Figure 1. [3]

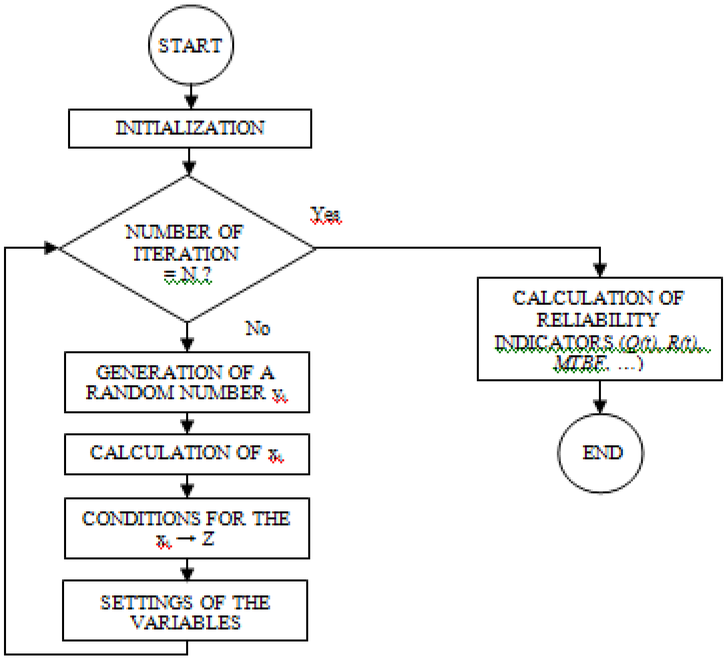

Figure 1: The algorithm of Monte Carlo analysis [6]

Step 1: Initialization - initialization includes a description of failure rate λ_i (or the other parameter) and probability distribution of each element, next the definition of number of iteration N and reset all variables.

Step 2: Generation of RNs and transformation - there are cyclically generated the random numbers y_i and calculated the values of x_i, via the transformation of x_i by (6). The transformation gives an equation in form $x_i = f\,(y_i)$. The number of operations generation-calculation of x_i depends on the number of system elements, i.e. if the system consists of j elements, then are generated j random numbers ($y_{i1}, y_{i2}, ..., y_{ij}$) and is made their transformation to $x_{i1}, x_{i2}, ..., x_{ij}$.

Step 3: Conditions for the $x_i \rightarrow Z$ – these conditions are used to creating of system mathematical model. In the case of a serial system, the z_i = min (x_{ij}). For the parallel system z_i = max (x_{ij}). There z_i is a mean time between failures for ***i***-th iteration, the $MTBF_i$.

Step 4: Settings of the variables - in this step are set the variables for the calculation of reliability indicators. These variables are defined in the Step 1.

Step 5: Calculation of reliability indicators - After N iterations are calculated the reliability indicators of the system. [6]

3. THE EXAMPLE OF MONTE CARLO RELIABILITY ANALYSIS

In this chapter is showed as example of using Monte Carlo simulation as a reliability analysis. The task is calculation of the reliability indicators of the whole system - probability of reliability $R(t)$ and probability of failure $Q(t)$ for time t = 10.000 h and Mean time between failures $MTBF$.

3.1. DESCRIPTION OF THE ANALYZED SYSTEM

The system created by 3 elements – A, B, C, see on the Figure 2, is the aim of the reliability analysis. The graphs of probability density $f(x)$ of each element of the Figure 2 are only illustrative.

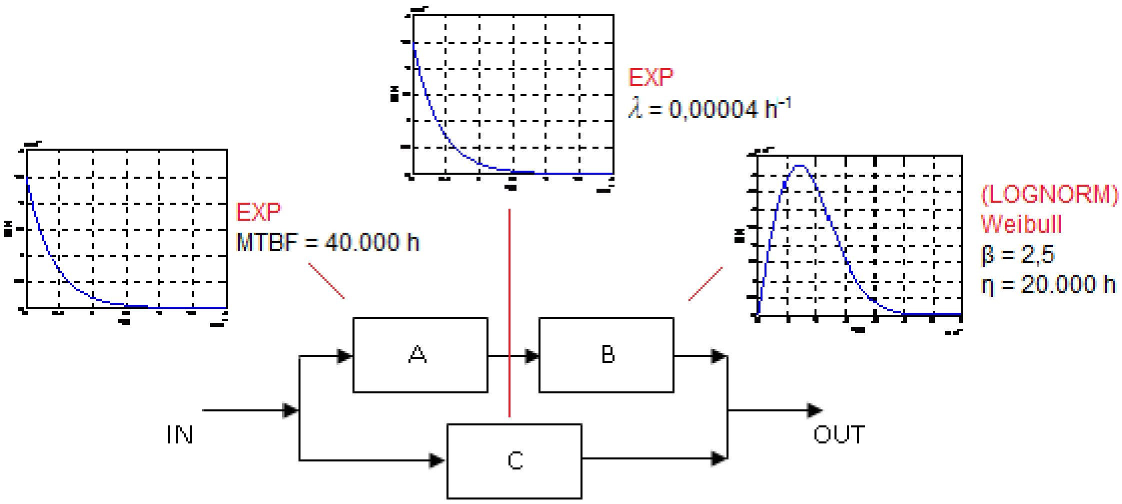

Figure 2: The reliability diagram of the analyzed system

The first component - A is a device with exponential distribution of probability and with Mean time between failures $MTBF_A$ = 40.000 h. In series with a device A is a component B. The component B has a lognormal distribution of probability. Its distribution was approximated by the 2-Parameter Weibull distribution with the parameters β_B = 2,5 and η_B = 20.000 h. In parallel to A and B is component C. This component (device) is the redundancy for the A and B. The device C has an exponential distribution of probability with a failure rate λ_C = 0,0001 h^{-1}.

3.2. RELIABILITY ANALYSIS

This analysis uses the algorithm showed on the Figure 1. There are described the individual steps of the algorithm applied to the given system on the Figure 2.

Step 1: Initialization

There are defined the reliability indicators and parameters and the other variables.

- Component A: failure rate: $\lambda_A = \dfrac{1}{MTBF_A} = \dfrac{1}{40.000} = 2{,}5 \cdot 10^{-5}\, h^{-1}$

- Component B: parameters: β_B = 2,5 and η_B = 20.000 h

- Component C: failure rate: λ_C = 0,0001 h^{-1}

Other variables: number of iteration: N = 1.000.000,

variable for the $MTBF$ calculation: S = 0,

variable for the $R(t)$ and $Q(t)$ calculation: K = 0

Step 2: RNs generation and transformation

The analyzed system contains of 3 elements (A, B, C). Therefore the process generation of y_i - transformation to x_i must be made 3-times, for each iteration *i*.

The exponential distribution of probability is expected for the components A and C. This distribution is described by the probability density *f*(*x*) via (2) [5], where λ is the failure rate. The component B is described by the 2-Parameter Weibull distribution of probability with the probability density *f*(*x*) by (3) [5], where η is the scale parameter (characteristic life) and β is the shape parameter.

$$f(x) = \lambda \cdot \exp(-\lambda x) \qquad [h^{-1}] \tag{2}$$

$$f(x) = \frac{\beta}{\eta} \cdot \left(\frac{x}{\eta}\right)^{\beta-1} \cdot \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right) \qquad [h^{-1}] \tag{3}$$

The equations (4) for A and C and (5) for B are given by the transformation via (1) and expression of x_i.

$$x_i = -\frac{1}{\lambda} \cdot \ln(1 - y_i) \tag{4}$$

$$x_i = \eta \cdot \left(-\ln(1 - y_i)\right)^{\frac{1}{\beta}} \tag{5}$$

The implementation of Step 2 is thus generation of y_{Ai}, y_{Bi}, y_{Ci} and transformation of the individual y_i to the x_{Ai}, x_{Bi}, x_{Ci} using the parameters λ or β and η.

Step 3: Conditions for the $x_i \rightarrow$ Z

The elements A and B are structured in the serial form, then z_{i_AB} = min (x_{Ai}, x_{Bi}). The whole system is a parallel combination of elements (A serial B) and C. The condition is thus z_i = max (z_{i_AB}, x_{Ci}).

Step 4: Settings of the variables

The settings of variables for *R*(*t*) (or *Q*(*t*)) and *MTBF* calculation occurs in this step. For the probability of reliability - *R*(*t*) calculation is the current z_i compared with a defined time *t* (there *t* = 10.000 *h*). Success means: $z_i > t$. In this case is the value of variable *K* increased by 1 (*K* = *K*+1).

For the mean time between failures - *MTBF* calculation is increased the value of variable *S* by z_i value ($S = S + z_i$).

Step 5: Calculation of the reliability indicators

The required reliability indicators *R*(*t*), *Q*(*t*) and *MTBF* are calculated after *N* iterations. The values of these indicators are given by (6), (7) and (8), where *K* and *S* are the program variables and *N* is the number of iterations.

$$R(t) = \frac{K}{N} \qquad [-] \tag{6}$$

$$Q(t) = 1 - R(t) \quad [-] \tag{7}$$

$$MTBF = \frac{S}{N} \quad [h] \tag{8}$$

The whole algorithm is implemented in MATLAB. The values of probability of reliability $R(t)$, probability of failure $Q(t)$ after 10.000 h and $MTBF$ of the analyzed system, obtained via Monte Carlo approach are (after N = 1.000.000 iterations):

$R(10.000\ h) = 0{,}7805$ $Q(10.000\ h) = 0{,}2195$ $MTBF = 17.183\ h$

The analytical expression of reliability indicators is quite difficult in case of the complex system with various distributions of probability of individual components. To verification of results this example is used software BlockSim 7.0 (trial version) by ReliaSoft. The results are:

$R(10.000\ h) = 0{,}7802$ $Q(10.000\ h) = 0{,}2198$ $MTBF = 17.179\ h$

4. CONCLUSION

The goal of this paper was to show, how to make the quantitative reliability analysis of the complex systems using Monte Carlo (MC) simulation. MC can be used to reliability analysis of systems with an arbitrary structure and with an arbitrary distribution of reliability of individual components. It is the main advantage of this approach. The whole algorithm of MC can be implemented in any programming languages with a good random number generator. There is a possibility to use commercial software, but its price is the main disadvantage. The use of MC is free, quite simple and the results are accurate.

In real terms, the results of the analysis can be used to provide economic justification for reliability improvements.

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REFERENCES

[1] Alexander, D.: Application of Monte Carlo Simulations to System Reliability Analysis [online]. Baton Rouge, Louisiana. Available on:

http://turbolab.tamu.edu/proc/pumpproc/P20/10.pdf

[2] Ericsson, C.A.: Hazard Analysis Techniques for System Safety. John Wiley & Sons, Inc., Hoboken, New Jersey 2005, ISBN 0-471-72019-4

[3] Singh, C., Mitra, J.: Monte Carlo Simulation for Reliability Analysis of Emergency and Standby Power Systems. Industry Applications Conference, 1995. Thirtieth IAS Annual Meeting, IAS '95, Conference Record of the 1995 IEEE. Volume 3. Pages 2290 – 2295.

[4] Smith, D., Simpson, K.: Functional Safety. Elsevier Butterworth-Heinemann, 2004. 258 s. ISBN 0-7506-6269-7

[5] Novotný, R.: Weibullovo rozdělení při analýzách bezporuchovosti [online]. Elektrorevue, 2002/17. Available on: http://www.elektrorevue.cz/clanky/02017/index.html

[6] JIRGL, M., Bradáč, Z., Stibor, K., Havlíková, M.: Reliability analysis of systems with a complex structure using Monte Carlo approach. In Programmable devices and systems. Velké Karlovice: Elsevier B.V., 2013. Pages 213-218. ISBN: 9783902823533. ISSN: 1474-6670.