SENSITIVITY ANALYSIS OF THE INDUCTION MACHINE SUBSTITUTING CIRCUIT ELEMENTS

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Abstract: Attainment of the identical shapes of measured and calculated torque characteristics of the induction machine is considerably difficult. This is due to implementation of a series of the simplifying assumptions when deriving the torque characteristics. During the derivation there are all parameters of the substituting circuit considered as constants. However, these assumptions do not apply to real machine. Especially change of the resistors values due to temperature rise during the measurement causes inaccuracy of characteristics comparison. The paper deals with the theoretically derived sensitivities of the torque characteristics for individual substituting circuit parameters.

Keywords: Sensitivity, analysis, substitute circuit, torque characteristics, induction machine

1. INTRODUCTION

It is known that attainment of the ideal coincidence between measured and calculated torque characteristics of the induction machine is considerably difficult. Deviations are caused by the change of the resistances due to skin effect and especially due to temperature rise of windings during the measurement. It is also known that these deviations cannot be explained just only by the change of resistances.

Therefore the sensitivity analysis of torque characteristics to individual elements of induction motor substituting circuit was performed.

2. SUBSTITUTING CIRCUIT OF THE INDUCTION MACHINE

The substituting circuit of the induction machine in the form of the Γ -network (see Figure 1) was chosen for the purpose of the sensitivity analysis. Purposely was not used T-network, because in [1] and [2] is shown that Γ -network is absolutely full-blown and equal when comparing it with T-network.



Figure 1: Substituting circuit of the induction machine in the form of the Γ -network.

The calculation of the sensitivity analysis will be based on the torque characteristics equation of the induction motor, which was derived for the Γ -network in [3]:

$$T = \frac{3U^{2}Rp}{\omega s} \frac{1}{\left[\left(1 + \frac{L_{2}}{L_{1}}\right)R_{1} + \left(1 + \frac{R_{1}}{R_{Fe}}\right)\frac{R}{s}\right]^{2} + \left[\left(1 + \frac{R_{1}}{R_{Fe}}\right)\omega L_{2} - \frac{R_{1}}{\omega L_{1}}\frac{R}{s}\right]^{2}}.$$
 (1)

In the displayed equation the following parameters occur: rotor resistance R, stator winding resistance R_1 , resistance R_{Fe} respecting iron losses, magnetizing inductance L_1 , leakage inductance L_2 . All of these parameters have its special influence on the final torque characteristics. The Table 1 shows values of these parameters for motor AOM 090L02-016, 2.2kW, 2p = 2.

R	R_1	$R_{ m Fe}$	L_1	L_2
2.245 Ω	2.910 Ω	982 Ω	0.387 H	0.019 H

Table 1:Substituting circuit parameters values.

3. SENSITIVITY ANALYSIS OF THE TORQUE CHARACTERISTICS

Sensitivity of the torque characteristics to the chosen substituting circuit parameter is calculated as the partial derivative of the toque characteristics, i.e. equation (1), according to the selected parameter.

3.1. SENSITIVITY OF CHARACTERISTICS TO THE ROTOR RESISTANCE R

It is appropriate to formally transform equation (1) into simplified shape

$$T = \frac{3U^2 ps}{\omega} \frac{R}{R^2 K_{1,R} + RK_{2,R} + K_{3,R}},$$
(2)

where $K_{1,R}$, $K_{2,R}$, $K_{3,R}$ are constants which arose from the modification of equation (1):

$$K_{1,R} = \left[\left(1 + \frac{R_1}{R_{\rm Fe}} \right)^2 + \frac{R_1^2}{\omega^2 L_1^2} \right], \qquad K_{2,R} = 2R_1 s \left(1 + \frac{R_1}{R_{\rm Fe}} \right), \qquad (3), (4)$$

$$K_{3,R} = \left[R_1^2 s^2 \left(1 + \frac{L_2}{L_1} \right)^2 + \omega^2 L_2^2 s^2 \left(1 + \frac{R_1}{R_{\text{Fe}}} \right)^2 \right].$$
(5)

Sensitivity to the change of the resistance R is determined as the derivative of equation (2) by the variable R:

$$\frac{\partial T}{\partial R} = \frac{3U^2 ps}{\omega} \frac{K_{3,R} - R^2 K_{1,R}}{\left[R^2 K_{1,R} + R K_{2,R} + K_{3,R}\right]^2} \,. \tag{6}$$

The term $(\partial T/\partial R)$ can be called as the *absolute* sensitivity. The *absolute* sensitivities to another parameters, e.g. $(\partial T/\partial R_1)$, $(\partial T/\partial L_1)$, can be determined by the similar way. However, the problem arises in the case when we want to compare the individual *absolute* sensitivities mutually. This is the reason why we are interested in the *relative* sensitivity $\frac{\partial T}{\partial R}$, i.e. the sensitivity to the *relative* $\frac{\partial R}{R}$.

change $(\partial R/R)$ of the resistance *R*. We can obtain this sensitivity by the modification of equation (6):

$$\frac{\partial T}{\partial R/R} = \frac{\partial T}{\partial R}R = \frac{3U^2 ps}{\omega} \frac{K_{3,R} - R^2 K_{1,R}}{\left[R^2 K_{1,R} + R K_{2,R} + K_{3,R}\right]^2}R.$$
(7)

Equation (7) is graphically displayed in the Figure 2.

The figure shows that sensitivity is negative for the slip s = 0 to 0.34, i.e. to the critical torque. Therefore, in this area decreases torque if the rotor resistance increases.



Figure 2: Sensitivity of the torque characteristics to the relative change of the rotor resistance *R*.

In the following chapters 3.2 to 3.4 are similarly derived and displayed sensitivities to the remaining parameters R_1 , R_{Fe} , L_1 , L_2 . In each chapter there are showed final derived equations and their corresponding graphical dependences. The Figure 6 in the conclusion then graphically summarizes performed sensitivity analysis.

3.2. SENSITIVITY OF CHARACTERISTICS TO THE STATOR WINDING RESISTANCE R_1

Sensitivity to the relative change of the resistance R_1 :

$$\frac{\partial T}{\partial R_1} R_1 = -\frac{3U^2 R_{SP}}{\omega} \frac{2R_1 K_{1,R1} + K_{2,R1}}{\left[R_1^2 K_{1,R1} + R_1 K_{2,R1} + \omega^2 L_2^2 s^2 + R^2\right]^2} R_1,$$
(8)

where constants $K_{1,R1}$, $K_{2,R1}$ are shown in [3].



Figure 3: Sensitivity of the torque characteristics to the relative change of the resistance R_1 . Figure 3 shows that machine torque decreases if the stator resistance increases (due to increasing temperature). This negative effect is observable in the whole range of the slip *s*.

3.3. SENSITIVITY OF CHARACTERISTICS TO THE RESISTANCE $R_{\rm FE}$

Sensitivity to the relative change of the resistance $R_{\rm Fe}$:

$$\frac{\partial T}{\partial R_{\rm Fe}} R_{\rm Fe} = \frac{3U^2 R_{\rm Sp}}{\omega} \frac{R_{\rm Fe}^2 K_{2,R\rm Fe} + 2R_{\rm Fe} K_{1,R\rm Fe}}{\left[R_{\rm Fe}^2 K_{3,R\rm Fe} + R_{\rm Fe} K_{2,R\rm Fe} + K_{1,R\rm Fe}\right]^2} R_{\rm Fe}, \tag{9}$$

where constants $K_{1,RFe}$, $K_{2,RFe}$, $K_{3,RFe}$ are shown in [3].



Figure 4: Sensitivity of the torque characteristics to the relative change of the resistance $R_{\rm Fe}$.

Figure 4 shows that sensitivity to the respecting iron losses resistance is positive in the whole range of the slip. This means that machine torque increases if the R_{Fe} increases too. On the other hand it is obvious that sensitivity to the resistance R_{Fe} has no effect to the torque characteristics.

3.4. SENSITIVITY OF CHARACTERISTICS TO THE INDUCTANCES L_1, L_2

Sensitivity to the relative change of the magnetizing inductance L_1 :

$$\frac{\partial T}{\partial L_1} L_1 = \frac{3U^2 R_s p}{\omega} \frac{L_1^2 2R_1^2 L_2 s^2 + L_1 2K_{1,L1}}{\left[L_1^2 K_{2,L1} + L_1 2R_1^2 L_2 s^2 + K_{1,L1}\right]^2} L_1.$$
(10)

Sensitivity to the relative change of the leakage inductance L_2 :

$$\frac{\partial T}{\partial L_2}L_2 = -\frac{3U^2 R_{sp}}{\omega} \frac{2L_2 K_{1,L2} + \frac{2R_1^2 s^2}{L_1}}{\left[L_2^2 K_{1,L2} + L_2 \frac{2R_1^2 s^2}{L_1} + K_{2,L2}\right]^2}L_2.$$
(11)

Constants $K_{1,L1}$, $K_{2,L1}$, $K_{1,L2}$, $K_{2,L2}$ are shown in [3].



Figure 5: Sensitivity of the torque characteristics to the relative change of the magnetizing inductance L_1 a), and of the leakage inductance L_2 b).

Figure 5 a) shows that inductance L_1 has no effect to the torque characteristics. Sensitivity to the magnetizing inductance parameter is positive in the whole range of the slip *s*. The rise of the mag-

netizing inductance causes the growth of the torque. At the same time this sensitivity is much smaller than sensitivity to the stator or rotor resistance.

Figure 5 b) shows that sensitivity to the leakage inductance parameter is negative in the whole range of the slip *s*. At the same time it is obvious that sensitivity to the leakage inductance L_2 is the greatest of all.

4. CONCLUSION

In the previous chapters 3.1 to 3.4 were derived and displayed sensitivities to the parameters R, R_1 , R_{Fe} , L_1 , L_2 of the substituting circuit of the induction machine. The results are summarized and compared in the Figure 6.

It is well known that deviations between measured and calculated torque characteristics cannot be explained just only by the change of the stator and rotor resistances. Presented analysis confirms the characteristics sensitivity to both resistances.

Additionally, it is very interesting that the torque characteristics is strongly dependent on the leakage inductance. Therefore, it is possible that the change of the leakage inductance can be responsible for the characteristics deviations. This possibility will be studied in the following experiments.





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