NUMERICAL ASPECTS OF INERTIAL NAVIGATION

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Abstract: This paper presents an investigation of the possibility of using the fixed-point arithmetic in the inertial navigation systems. Two square root filtering methods are copared with respect to conventional Kalman filter and its Joseph's stabilized form. Main contribution of this research lies in the evaluation of the minimal effective fixed point arithmetic word length for the pressented Phi angle error model with considered noise statistics.

Keywords: Inertial navigation, multisensor data fusion, Kalman filtering, square root filtering, fixed-point arithmetics, Phi angle error model, 15-state navigation algorithm

1 INTRODUCTION

The task related to the numerical difficulties of the conventional Kalman filter was investigated many times over past years. Special attention was paid to the spacecraft navigation and orbit determination problems as can be seen for example in [1]. The navigation model presented here, which is commonly used as a core of today's aircraft navigation algorithms, was not investigated yet (to the best authors' knowledge). This is probably due to using the floating-point arithmetic in the navigation computers. However, if we have an application, where using of the fixed-point arithmetic can be advantageous (e.g. systolic array implementation in the FPGA), than this work can be useful.

2 INERTIAL NAVIGATION ALGORITHM

The inertial navigation algorithm uses four reference frames. These are the inertial frame, earth frame, local navigation frame(here expressed in the ENU form) and body frame. By using the relations between these frames we can derive equations which describes the position, velocity and attitude of a moving target as follows (the time indexes are omitted due to simplification of the notation)

$$\dot{r}_n = T v_{en}^n \tag{1}$$

$$\dot{v}_{en}^n = C_b^n f_{ib}^b - (\Omega_{en}^n + 2\Omega_{ie}^n) v_{en}^n + g^n \tag{2}$$

$$\dot{C}_b^n = C_b^n \Omega_{ib}^b - (\Omega_{ie}^n + \Omega_{en}^n) C_b^n \tag{3}$$

$$\dot{b}_a = \nu_{ba} \tag{4}$$

$$\dot{b}_g = \nu_{bg} \tag{5}$$

where $r_n = [L\, \lambda\, h]$ is the position of the local navigation frame origin and $v_{en}^n = \left[v_{en,E}^n \; v_{en,N}^n \; v_{en,U}^n\right]$ is its velocity. T is the transformation matrix which transforms vectors from the local navigation frame to the curvilinear coordinates. C_b^n is the direction cosine matrix expressed in terms of the roll, pitch and yaw angles. This matrix transforms the body frame expressed vectors into the navigation frame. The terms b_a and b_g each consist three scalars which represents accelerometers' and gyroscopes' biases respectively. They are modelled as the Brownian motion plus a random constant and are used to correct the accelerometer and gyroscope measurements. The term f_{ib}^b represents the accelerometer measurement vector and the term Ω^b_{ib} is the skew-symmetric matrix of the gyroscope measurements. Ω^n_{ie} is the skew-symmetric matrix of the Earth's angular rate and Ω^n_{en} represents the skew-symmetric matrix of the angular rate due to translational motion. Finally the term g^n is the local gravity vector.

For the navigation equations (1)-(5) can be derived a corresponding error state space model by using the linear perturbation analysis. This error model is used in the indirect Kalman filter and estimated state vector is used for correcting the inertial navigation system's solution which is provided by integrating the equations (1)-(5) through time. If this solution is not corrected than the navigation system integrates noise given by the gyroscope and accelerometer measurements which results in the divergence and failure of the algorithm. Resulting error model is represented by the continuous linear time variant stochastic system which contains 15 error states as follows

$$\begin{bmatrix} \delta\dot{r}_n \\ \delta\dot{v}^n_{en} \\ \dot{\phi} \\ \delta\dot{b}_a \\ \delta\dot{b}_g \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & O_3 & O_3 & O_3 \\ F_{21} & F_{22} & F_{23} & C^n_b & O_3 \\ F_{31} & F_{32} & F_{33} & O_3 & -C^n_b \\ O_3 & O_3 & O_3 & O_3 & O_3 \\ O_3 & O_3 & O_3 & O_3 & O_3 \end{bmatrix} \begin{bmatrix} \delta r_n \\ \delta v^n_{en} \\ \phi \\ \delta b_a \\ \delta b_a \end{bmatrix} + \begin{bmatrix} O_3 & O_3 & O_3 & O_3 \\ C^n_b & O_3 & O_3 & O_3 \\ O_3 & -C^n_b & O_3 & O_3 \\ O_3 & O_3 & I_3 & O_3 \\ O_3 & O_3 & O_3 & I_3 \end{bmatrix} \begin{bmatrix} \nu_a \\ \nu_g \\ \nu_{ba} \\ \nu_{bg} \end{bmatrix} \quad (6)$$

All terms are intuitively related as the errors with respect to equations (1)-(5), the three component vector ϕ represents the errors in the roll, pitch and yaw angles. Each sub-matrix F are of the dimension $3x3$ and represents the state space transition matrix. The terms ν_a and ν_g are the accelerometers' and gyroscopes' measurement noises. Further, ν_{ba} and ν_{bg} are their corresponding bias noises. All of these are assumed as a zero mean uncorrelated Gaussian white noises with known covariances.

The measurement model is expressed as

$$\delta z = H\delta x + \mu \quad (7)$$

where the term $\mu = [\mu_r \ \mu_v]^T$. μ_r and μ_v are the GNSS position and velocity noises respectively, which are again assumed as a zero mean uncorrelated Gaussian white noises with known covariances.

$$H = \begin{bmatrix} T & O_3 & O_3 & O_3 & O_3 \\ O_3 & I_3 & O_3 & O_3 & O_3 \end{bmatrix} \quad (8)$$

The input measurements for the indirect Kalman filter update residuum are expressed as the difference between the GNSS observed position and velocity and the navigation system's solution given by the equations (1) and (2). More details about INS algorithm can be found in [2].

3 SIMULATION RESULTS

Let us assume that we have a vehicle which moves in the circular trajectory with constant ground speed 10m.s^{-1} and at constant altitude 1000m. The accelerometers' noise std. deviations are given as $\sigma_a = 9.81\times10^{-5}\text{m.s}^{-3/2}$ and $\sigma_{ba} = 6.00\times10^{-5}\text{m.s}^{-5/2}$. For the gyroscopes' std. deviations we have $\sigma_g = 2.91\times10^{-7}\text{rad.s}^{-1/2}$ and $\sigma_{bg} = 9.20\times10^{-7}\text{rad.s}^{-3/2}$. The accelerometers' biases are all three set to -0.03m.s^{-2} and gyroscopes' biases to $2.95\times10^{-4}\text{rad.s}^{-1}$, where first one of this gyro bias have negative sign. The GNSS position and velocity measurements have std. deviations $\sigma_r = 10$m and $\sigma_v = 4\text{m.s}^{-1}$ respectively. The accelerometers' and gyroscopes' measurements are both sampled every 0.2s and GNSS measurements every 1s. Simulation is performed in the MATLAB/Simulink environment with its Fixed-Point Toolbox. The fixed-point arithmetic has 46 bits in the integer part and 45 bits in the fractional part. The length of the simulation is 1000s.

Due to 3 page limit for this publication, there is no possible to depict all 15 estimated states, so the figure (1) shows only estimates of the altitude (a), east velocity (b), yaw angle (c) and accelerometers' x-axis bias (d). Each of these shows GNSS measurements (expect for yaw angle and bias, because they can not be observed by the GNSS), estimates of the conventional, Joseph's, Potter square root and UD decomposed Kalman filters computed in fixed-point (fxp) arithmetic with previously noted parameters. All these Kalman filter forms can be found in [3]. The conventional Kalman filter computed in floating-point (flp) double precision arithmetic is shown for comparison. As can be seen from picture (1a) the conventional and Joseph's algorithms diverge after approximately $50s$. This is caused in the Kalman filter time update step, where the multiplication of the term $\Gamma Q \Gamma^T$ ([1] p.28) is performed with rounding some elements to zero (Γ is the matrix of the second term of the equation (6) and Q is the process noise covariance matrix related to the noise terms in (6)). This leads to the situation, when the Kalman filter stops forgetting old data. Measurement update and the system transition matrix causes that the covariance matrix eigenvalues converges to zero which forces this matrix to be ill-conditioned (infinite condition number), than estimated altitude error diverges and altitude as the navigation algorithm state variable too. This phenomena can be overcome with using the square root filters as figure (1a) shows. Similar conclusions can be made with rest of the depicted variables.

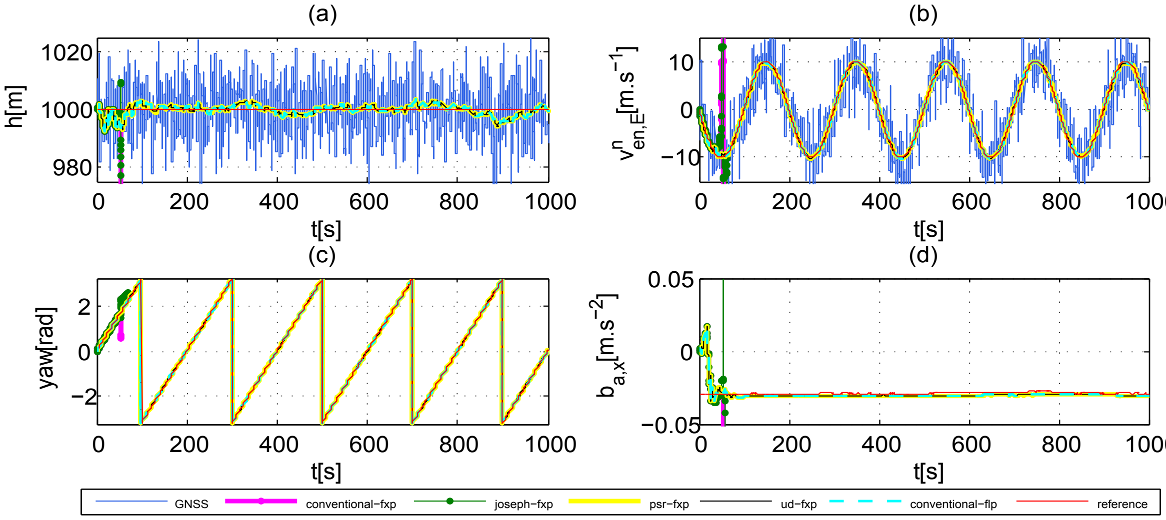

Figure 1: Estimated altitude (a), east velocity (b), yaw angle (c) and accelerometers' x-axis bias (d) for the various Kalman filter implementations.

4 CONCLUSION

This work showed, how the numerical performance of the conventional Kalman filter, applied to the inertial navigation, can be improved by using the covariance matrix factorizations. The phenomena of the rounding errors which can cause a divergence of the conventional and Joseph's form Kalman filters was described on the example, where a vehicle moves in the circular trajectory.

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