SIMULATION AND VERIFICATION OF METHODS FOR PARTIAL DISCHARGE SOURCE LOCALIZATION

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Abstract: This article deals with possibilities of localization of partial discharges (PD) in oil power transformers. Localization can be performed by means of analysis of UHF waveforms which are measured during activity of the partial discharges. The time differences of arrival (TDOA) of the waveforms related to transient process occurrence in the signals are the main input parameters for localization methods. In order to estimate the position of the signal source in 3D space a minimum of four antennas has to be used, since the time of the PD occurrence is unknown. For localization purposes the system of nonlinear equations has to be solved. This can be performed by means of analytical or numerical methods. Both of the methods have their own advantages and disadvantages. The application of both approaches for signal source localization in measurement model will be presented in this paper. The methods will be compared in view of their suitability for a given configuration of antenna system.

Keywords: partial discharges, TDOA

1. INTRODUCTION

In order to prevent the transformer failure the observation of pulse activity of PD is necessary. The occurrence of discharge with substantial charge transport level can be localized in critical areas of the transformers. Having possibility to localize the increased discharge activity in some of critical areas allows undertaking precautions in order to avoid the critical transformer failure. The discharge activity localition can be estimated by processing of signals from suitable installed sensors [1].

2. PRINCIPLE OF LOCALIZATION

In case when sensors are realized as antennas, it is possible to determine the TDOA from antennas output signals. For this situation, the propagation time of the signal from source of discharge activity to antenna, which is closest, is unknown. The TDOA of signals arriving on each of antennas can be determined only. Therefore, there are four unknowns, source coordinates *x*, *y*, *z* and propagation time of the signal from the source to the first antenna t_0. Generally equations for n^{th} antenna is

$$(x-x_n)^2+(y-y_n)^2+(z-z_n)^2=\left(\frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}}(t_0+t_{1n})\right)^2, \tag{1}$$

where x_n, y_n, z_n are coordinates of n^{th} antenna and t_{1n} is TDOA between the first and n^{th} antenna, $t_{1n} = t_1 – t_n$. On the base equations (1) is possible to write a system of four equations in order to found four unknowns

$$\left(x-x_1\right)^2+\left(y-y_1\right)^2+\left(z-z_1\right)^2=\left(\frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}}t_0\right)^2, \tag{2}$$

$$\left(x-x_2\right)^2+\left(y-y_2\right)^2+\left(z-z_2\right)^2=\left(\frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}}\left(t_0+t_{12}\right)\right)^2, \tag{3}$$

$$\left(x-x_3\right)^2+\left(y-y_3\right)^2+\left(z-z_3\right)^2=\left(\frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}}\left(t_0+t_{13}\right)\right)^2, \tag{4}$$

$$\left(x-x_4\right)^2+\left(y-y_4\right)^2+\left(z-z_4\right)^2=\left(\frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}}\left(t_0+t_{14}\right)\right)^2, \tag{5}$$

Position of the source is determined by solution of equation system (2)-(5), where defined antennas coordinates and measured TDOA parameters are used.

The system (2)-(5) is a non-linear equation system. This system can be solved by numerical or analytical methods. The basic numerical method for non-linear equation system solution is the Newton's method. This method and other numerical methods are based on finding of roots of non-linear vector equations

$$\mathbf{F}\left(\mathbf{X}\right)=0, \tag{6}$$

where **F**(**X**) is non-linear vector function which components are given by system (2)-(5). The components are modified into homogenous form. **X** is a vector variable which components consist of *x*, *y*, *z*, t_0. Vector function **F**(**X**) is possible to expand according to Taylor's series in vicinity of root iteration $\mathbf{X}_n$

$$\mathbf{F}\left(\mathbf{X}_{n+1}\right)=\mathbf{F}\left(\mathbf{X}_n\right)+\frac{\partial\mathbf{F}\left(\mathbf{X}_n\right)}{\partial\mathbf{X}}\left(\mathbf{X}_{n+1}-\mathbf{X}_n\right)+\frac{1}{2}\frac{\partial^2\mathbf{F}\left(\mathbf{X}_n\right)}{\partial\mathbf{X}^2}\left(\mathbf{X}_{n+1}-\mathbf{X}_n\right)^2. \tag{7}$$

The equation (7) can be reduced to first order

$$\mathbf{F}\left(\mathbf{X}_{n+1}\right)=\mathbf{F}\left(\mathbf{X}_n\right)+\frac{\partial\mathbf{F}\left(\mathbf{X}_n\right)}{\partial\mathbf{X}}\left(\mathbf{X}_{n+1}-\mathbf{X}_n\right). \tag{8}$$

The solution of the vector equation is achieved by means of iterative procedure. The starting point in equation (8) is estimation of vector $\mathbf{X}_0$. The following root iterations are determined by

$$\mathbf{X}_{n+1}=\mathbf{X}_n+\mathbf{D}_n=\mathbf{X}_n-\frac{\mathbf{F}\left(\mathbf{X}_n\right)}{\dfrac{\partial\mathbf{F}\left(\mathbf{X}_n\right)}{\partial\mathbf{X}}}, \tag{9}$$

where

$$\mathbf{J}\left(\mathbf{X}_n\right)=\frac{\partial\mathbf{F}\left(\mathbf{X}_n\right)}{\partial\mathbf{X}}, \tag{10}$$

is a Jacob's matrix (Jacobian) of equation system in point $\mathbf{X}_n$. A modification of the Newton's method or other methods for solution of non-linear equation system can be used also.

The second approach to localization problem exploits analytical solution of the non-linear equation system. The analytic methods are based on direct solution of non-linear set of equations. The important issue in the analytical solution of system (2)-(5) is the non-symmetry due to character of equation (1). However, it is possible to overcome this by suitable linear combination of equations containing time of arrival parameters only (TOA) [3].

3. NUMERICAL METHOD APPLICATION

The application of numerical method for source localization has been experimentally examined on a measurement model. The model consists of four receiving antennas connected to acquisition device and one pulsed signal source. The arrangement of the model is shown in Fig. 1A). The antennas positions are marked as A1, A2, A3 and A4. The antennas arrangement has been chosen similar to the arrangement in target application. The given arrangement is determined by construction possibilities of the transformer. The pulsed signal source has been placed in position marked as 1, 2, 3, ..., 7. The coordinates system in relation to the antennas positions is shown in Fig. 1B). The coordinates of antennas in direction *x* and *z* are $x = 0$ and $z = 0$. The TDOA parameters between acquired signals from antennas A1, A2, A3 and A4 have been determined by means of threshold crossing algorithm [4]. Further, the TDOA parameters have been forwarded to localization algorithm. In order to source position determination a Newton's method has been chosen. This method is included in numerical tools of Optimization Toolbox in MATLAB system. The Newton's method is utilized in case of Large-scale problems solution. In first computation trials the algorithm wasn't converging. Large order difference between the input parameters has been suspected as one of the reasons. Since the propagation velocity is in order of 10^8 m/s and the TDOA values are in order of 10^{-9} s, the order difference is 10^{17}, which is an extreme value. While the numerical methods are using finite numerical steps between the iterations, the extreme value of order difference may cause the algorithm to fail. Therefore the reduction of the order difference has been proposed in order to reduce this extreme value. Using this reduction no influence on the results should occur. The reduced order difference which has been found for stable and fast convergence was 10^{11}. The values of the TDOA for signal positions 1, 3, 4, 6, 7 only have been processed by MATLAB algorithm due to symmetry of the measurement model. The exact results have been found for positions 1 and 3 only. Therefore, another order differences have been examined. It has been found that order difference 10^{-1} provided convergence for almost all source positions 1, 3, 4, 6. However the algorithm wasn't converging for source position 7. It is evidently caused by poor geometric dilution of precision for this position. Based on the results of numerical method verification it may be concluded that its application is potentially problematic due to convergence problem. The significant factor on the convergence and results precision is antennas arrangement, which was subject to transformer construction possibilities.

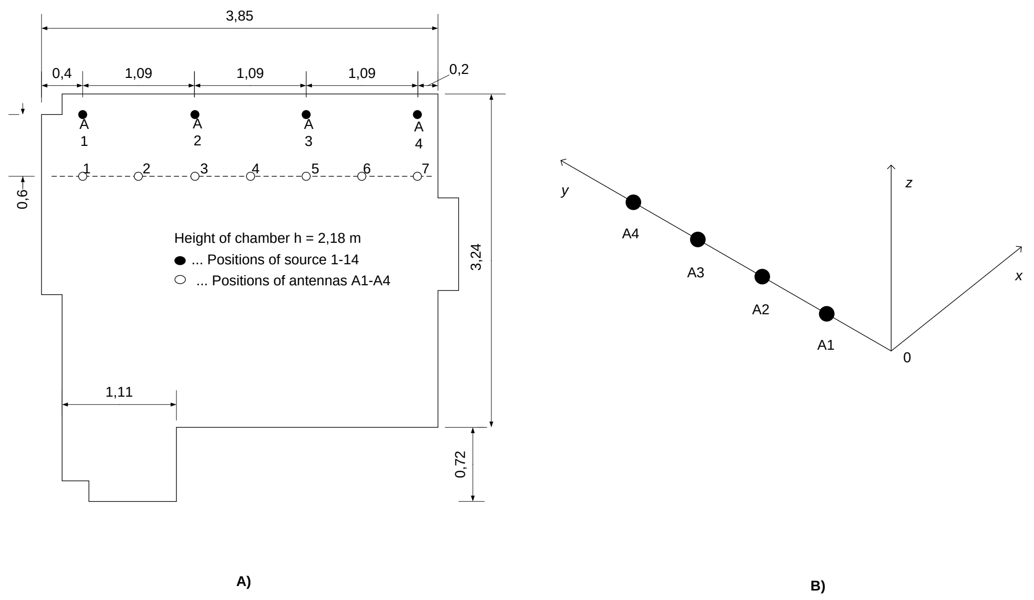

Figure 1: A) The antennas positions in measurement model, B) system of coordinates related to antennas positions

4. ANALYTICAL METHOD APPLICATION

An example approach to analytical solution of non-linear set of equations has been published in [3]. The approach is based on subtraction of TOA equations which are similar to (2)-(5). Therefore the TDOA parameters can be used instead of the TOA parameters and the set symmetry is provided also.

Due to given antennas arrangement the solution and therefore the source position is ambiguous. For a given set of TDOA parameters the solution (source position) lies on circle, which's plane is perpendicular to the straight line join of antennas. Therefore the solution in 2D space may be sought only. Then, the source of signal may be found as an intersection of the circle with relevant part of inner transformer structure. A new proposed method suppose the source position in plane xy and the position of the first antenna is $x_1 = 0$, $y_1 = 0$. The initial equation system defines source position towards three antennas

$$x^2 + y^2 = v^2 t_0^{\ 2}, \tag{11}$$

$$x^2 + (y - y_2)^2 = v^2 (t_0 + t_{12})^2, \tag{12}$$

$$x^2 + (y - y_3)^2 = v^2 (t_0 + t_{13})^2, \tag{13}$$

where v is the propagation velocity and t_{ij} are input TDOA parameters. By direct solution of equation set (11)-(12) it is possible to deduce solution t_0, y, x

$$t_0 = \frac{v^2 \left(y_2 t_{13}^{\ 2} - y_3 t_{12}^{\ 2} \right) + y_3 y_2^{\ 2} - y_2 y_3^{\ 2}}{2v^2 \left(y_3 t_{12} - y_2 t_{13} \right)}, \tag{14}$$

$$y = \frac{y_2}{2} - \frac{v^2 \left(y_2 t_{13}^{\ 2} - y_3 t_{12}^{\ 2} \right) + \left(y_3 y_2^{\ 2} - y_2 y_3^{\ 2} \right)}{2 y_2 \left(y_3 t_{12} - y_2 t_{13} \right)} t_{12} - \frac{v^2 t_{12}^{\ 2}}{2 y_2}, \tag{15}$$

$$x = \sqrt{v^2 \left(\frac{v^2 \left(y_2 t_{13}^{\ 2} - y_3 t_{12}^{\ 2} \right) + y_3 y_2^{\ 2} - y_2 y_3^{\ 2}}{2v^2 \left(y_3 t_{12} - y_2 t_{13} \right)} \right)^2 - \left(\frac{y_2}{2} - \frac{v^2 \left(y_2 t_{13}^{\ 2} - y_3 t_{12}^{\ 2} \right) + \left(y_3 y_2^{\ 2} - y_2 y_3^{\ 2} \right)}{2 y_2 \left(y_3 t_{12} - y_2 t_{13} \right)} t_{12} - \frac{v^2 t_{12}^{\ 2}}{2 y_2} \right)^2}\ . \tag{16}$$

source position	time difference of the signals [ns]		analytical solutions						mean of the solutions		real position of the source [m]	
			p1 [m] (t12; t13)		p2 [m] (t13; t14)		p3 [m] (t12; t14)		p [m]			
			x	y	x	y	x	y	x	y	x	y
1	t_{12}	2,2										
	t_{13}	5,5	0,7895	-0,1	0,661	-0,056	0,359	0,167	0,6	-0,009	0,6	0
	t_{14}	9,1										
3	t_{12}	-2,2										
	t_{13}	0	0,5713	1,09	0,574	1,092	0,59	1,09	0,58	1,091	0,6	1,09
	t_{14}	3,4										
4	t_{12}	-3,1										
	t_{13}	-3,1	0,605	1,64	0,605	1,635	0,605	1,635	0,61	1,635	0,6	1,64
	t_{14}	0										
6	t_{12}	-3,5										
	t_{13}	-6,6	0,5922	2,71	0,597	2,723	0,603	2,725	0,6	2,718	0,6	2,73
	t_{14}	-6,6										
7	t_{12}	-3,6										
	t_{13}	-6,9	0,2499	2,37	0,359	3,103	0,661	3,326	0,42	2,932	0,6	3,27
	t_{14}	-9,1										

Table 1. Comparison of analytically calculated and real source coordinates

It is possible to found the source position using (14)-(15) when antennas coordinates y_2, y_3 , propagation velocity v and TDOA parameters t_{12}, t_{13} are known. Results of analytical method application are presented in Table 1. The analytical solutions p1, p2 and p3 in Table 1 are valid for various combinations of 4 antennas (only 3 antennas are used). The calculated results of the source positions are in a good accordance to the real source positions. In case of position 7 the accuracy is poor, probably to poor geometric dilution of precision.

5. CONCLUSION

In this paper the verification of methods for partial discharge source localization has been presented. The methods have been applied on specific antennas arrangement which is identical to arrangement of antennas in target application – 300 MVA high voltage power transformer [5]. The numerical and analytical methods have been briefly described and applications of these methods have been shown. An approach based on analytical method for discharge activity localization in transformer seems to be more suitable than numerical method. This conclusion is based on better accordance of analytical results to the real source positions. However, this is given by specific antennas arrangement, which doesn't dispose with a significant geometric dilution of precision. Another advantage is the calculation time, which is generaly shorter in compare to numerical methods.

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