OBSERVER BASED ADAPTIVE NONLINEAR CONTROL SYSTEM DESIGN FOR A SINGLE LINK MANIPULATOR

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Abstract: The paper presents the implementation of the interlaced controller-observer design approach for nonlinear adaptive trajectory tracking control of DC motor driven single link industrial manipulator. The nonlinear closed-loop adaptive system can track asymptotically a desired reference trajectory without angular speed measurement. Its performance is illustrated via dynamic simulation.

Keywords: Adaptive nonlinear control systems, Adaptive nonlinear observers, Trajectory tracking.

1. INTRODUCTION

There are several substantial considerations that condition contemporary control system design. On the first place the real physical systems are inherently nonlinear. Hence, the ever increasing need for high precision control demands special nonlinear design approaches. The second consideration is that most control system design methods are based on a mathematical model of the objective system. If the model fails to describe the system behaviour equivalently then the respective method of control also fails to meet the required quality performance specifications because the equivalence property is absolutely necessary and this is true for all methods of dynamic control. But actually, the model parameters are generally unknown and/or changing dynamically, so some type of adaptation mechanism has to be used to compensate for these uncertainties. On the third place, the high performance of the control process must be guaranteed in the entire range of operation which again implies the use of advanced nonlinear control system design approaches. Different methods have been proposed [1], [3], [6] based on Lyapunov theory. Most of them, such as adaptive backstepping, tuning functions, combined error, nonlinear damping, different types of prediction error based parameter estimators, require measurement of the whole state vector. Some of the parameter estimators based adaptive approaches provide asymptotic stability of the estimators but this is a trade off with the persistent excitation signal needed. Others provide only Lyapunov stability of the estimates which in general leads to inaccurate estimation of the unknown parameters. Other methods are built on output feedback only and use a special nonlinear observer for estimation of the state vector necessary for the nonlinear state feedback. Both groups provide asymptotic stability of the closed-loop adaptive system.

This paper is based on a previous work in the field of nonlinear adaptive control [4], [5] and presents the implementation of the interlaced controller-observer design approach [2], for nonlinear adaptive trajectory tracking control system design of a DC motor driven single link manipulator. The dynamics of the objective nonlinear system is decomposed into mechanical and electrical. The second order mechanical part has the motor current as input. The electrical part describes the dynamics of the DC motor with the motor voltage as input. In that sense, the above mentioned design approach is used for synthesis of a control law considering only the mechanical part. It defines the necessary current of the motor, which is achieved via current tracking PI controller. In the design all system parameters are considered as unknown and the angular position and the motor current are assumed to be measured.

2. INTERLACED ADAPTIVE CONTROLLER-OBSERVER DESIGN APPROACH

The interlaced controller-observer design approach is presented in its general form for single input-single output systems of second order transformed into the output feedback canonical form

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \boldsymbol{\varphi}_0(y) + \sum_{j=1}^{q} a_j \boldsymbol{\varphi}_j(y) + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} \sigma(y)u, \ y = \mathbf{C}\mathbf{x}, \qquad (1)$$

where $\mathbf{C} = \mathbf{e}_1^T = [1,0]$, $\mathbf{e}_2^T = [0,1]$, $\mathbf{x} = [x_1, x_2]^T$ is the state vector with x_2 – unmeasurable, u is the input, y is the output, $\boldsymbol{\varphi}_j(y) = [\varphi_{j1}(y), \varphi_{j2}(y)]^T$, $0 \le j \le q$ and $\sigma(y)$ are smooth nonlinear functions, and $\mathbf{a} = [a_1, \ldots, a_q]^T$, b_0 are unknown constant parameters. The approach requires the sign of b_0 to be known and $\sigma(y) \neq 0 \ \forall y \in \Re$. The design objective is to make the output of the system $y(t)$ track a reference trajectory $y_r(t)$ with known and bounded derivatives. The method replaces the unmeasured state variables with parameter dependant estimates. The observer error and the filter equations

$$\boldsymbol{\varepsilon} \doteq \mathbf{x} - (\boldsymbol{\xi}_0 + \sum_{j=1}^{q} a_j \boldsymbol{\xi}_j + b_0 \boldsymbol{\upsilon}_0), \qquad (2)$$

$$\dot{\boldsymbol{\xi}}_0 = \mathbf{A}_0 \boldsymbol{\xi}_0 + \mathbf{k}y + \boldsymbol{\varphi}_0(y) + \mathbf{n}, \ \dot{\boldsymbol{\xi}}_j = \mathbf{A}_0 \boldsymbol{\xi}_j + \boldsymbol{\varphi}_j(y), \quad 1 \le j \le q, \ \dot{\boldsymbol{\upsilon}}_0 = \mathbf{A}_0 \boldsymbol{\upsilon}_0 + \mathbf{e}_2 \sigma(y)u \qquad (3)$$

are defined. Considering (1), (2) and (3) the observer error dynamics can be computed as $\dot{\boldsymbol{\varepsilon}} = \mathbf{A}_0 \boldsymbol{\varepsilon} + \mathbf{n}$, where $\mathbf{n}$ is a correction term to be designed and $\mathbf{k} = [k_1, k_2]^T$ is chosen so that $\mathbf{A}_0 = \mathbf{A} - \mathbf{k}\mathbf{C}$ is Hurwitz. Considering (2) the unmeasured state variable x_2 is replaced with its estimate $x_2 = \varepsilon_2 + \xi_{02} + \boldsymbol{\xi}_{(2)} \mathbf{a} + \upsilon_{02} b_0$ into the dynamics of the tracking error $z_1 = y - y_r$, which then reads

$$\dot{z}_1 = -c_1 z_1 + b_0 z_2 + b_0(\alpha_1 + \hat{p}\Psi) + b_0 \tilde{p}\Psi + [\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, 0]\tilde{\boldsymbol{\theta}}] + \varepsilon_2, \qquad (4)$$

with $c_1 > 0$, $\boldsymbol{\xi}_{(2)} = [\xi_{12}, \ldots, \xi_{q2}]$, $\boldsymbol{\varphi}_{(1)} = [\varphi_{11}, \ldots, \varphi_{q1}]$ and $\Psi = c_1 z_1 + \xi_{02} + \varphi_{01} + [\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, 0]\hat{\boldsymbol{\theta}} - \dot{y}_r$. In (4) a new error variable $z_2 = \upsilon_{02} - \alpha_1$ is defined with υ_{02} as a virtual control, α_1 as a stabilizing function and $p = b_0^{-1}$ as a new unknown parameter. The function α_1 has to stabilize (4) with respect to the Lyapunov function candidate

$$V_1 = (1/2)z_1^2 + (1/2)\tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{\theta}} + (|b_0|/2\gamma_p)\tilde{p}^2 + (1/2d_1)\boldsymbol{\varepsilon}^T \mathbf{P}_0 \boldsymbol{\varepsilon}, \qquad (5)$$

where $\boldsymbol{\Gamma}_{(q+1)x(q+1)}$ is a positive definite matrix, $\gamma_p > 0$, $d_1 > 0$ and $\mathbf{P}_0$ is the positive definite solution of the Lyapunov equation $\mathbf{A}_0^T \mathbf{P}_0 + \mathbf{P}_0 \mathbf{A}_0 = -2\mathbf{I}$. The time derivative of (5) with respect to (4) reads

$$\begin{aligned} \dot{V}_1 = &-c_1 z_1^2 + b_0 z_1 z_2 + b_0 z_1(\alpha_1 + \hat{p}\boldsymbol{\Psi}) - d_1^{-1}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} + (|b_0|/\gamma_p)[\gamma_p \mathrm{sign}(b_0) z_1 \boldsymbol{\Psi} - \dot{\hat{p}}]\tilde{p} + \\ &+ \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1}\{\boldsymbol{\Gamma}[\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, 0]^T z_1 - \dot{\hat{\boldsymbol{\theta}}}]\} + d_1^{-1}\boldsymbol{\varepsilon}^T \mathbf{P}_0(d_1 \mathbf{P}_0^{-1}\mathbf{e}_2 z_1 - \mathbf{n}) \end{aligned} \qquad (6)$$

If υ_{02} is the actual control then $z_2 \equiv 0$ and with the choices

$$\alpha_1 = -\hat{p}\Psi, \quad \dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\tau}_1 = \boldsymbol{\Gamma}[\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, 0]^T z_1, \qquad \dot{\hat{p}} = \gamma_p \mathrm{sign}(b_0) z_1 \Psi, \qquad \mathbf{n} = \zeta_1 = d_1 \mathbf{P}_0^{-1}\mathbf{e}_2 z_1$$

the sign-indefinite terms in $\dot{V}_1$ are eliminated. Since υ_{02} is not the actual control $z_2 \neq 0$. We retain $\boldsymbol{\tau}_1$ as a tuning function, ζ_1 as a interlacing function and substituting α_1 and $\dot{\hat{p}}$ in (4) and (6) we obtain

$$\dot{V}_1 = -c_1 z_1^2 + b_0 z_1 z_2 - d_1^{-1}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1}(\boldsymbol{\tau}_1 - \dot{\hat{\boldsymbol{\theta}}}) + d_1^{-1}\boldsymbol{\varepsilon}^T \mathbf{P}_0(\zeta_1 - \mathbf{n}), \qquad (7)$$

$$\dot{z}_1 = -c_1 z_1 + b_0 z_2 + b_0 \tilde{p}\Psi + [\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, 0]\tilde{\boldsymbol{\theta}} + \varepsilon_2. \qquad (8)$$

The next step is to compute the time derivative of z_2

$$\dot{z}_2 \doteq \sigma(y)u + \beta_2 - \tilde{\boldsymbol{\theta}}^T(\partial\alpha_1/\partial y)\boldsymbol{\omega} - (\partial\alpha_1/\partial y)\boldsymbol{\omega}^T\hat{\boldsymbol{\theta}} - (\partial\alpha_1/\partial y)\varepsilon_2 - (\partial\alpha_1/\partial\hat{\boldsymbol{\theta}})\dot{\hat{\boldsymbol{\theta}}} - (\partial\alpha_1/\partial\boldsymbol{\xi}_0)\mathbf{n}\,, \quad (9)$$

where $\boldsymbol{\omega}^T = [\boldsymbol{\xi}_{(2)} + \boldsymbol{\varphi}_{(1)}, \boldsymbol{\upsilon}_{(2)}]$ and β_2 denotes the all known terms. The control u has to be designed to make the error system, described by (8) and (9), asymptotically convergent to zero. An augmented Lyapunov function candidate $V_2 = V_1 + (1/2)z_2^2$ is defined whose derivative is

$$\begin{aligned}\dot{V}_2 = -c_1z_1^2 - d_1^{-1}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}[\boldsymbol{\tau}_1 - \boldsymbol{\Gamma}\overline{\boldsymbol{\varpi}}z_2 - \dot{\hat{\boldsymbol{\theta}}}] + d_1^{-1}\boldsymbol{\varepsilon}^T\mathbf{P}_0[\boldsymbol{\zeta}_1 - \mathbf{n} - d_1\mathbf{P}_0^{-1}(\partial\alpha_1/\partial y)z_2\mathbf{e}_2] + \\ + z_2[\sigma(y)u + \beta_2 - \overline{\boldsymbol{\varpi}}^T\hat{\boldsymbol{\theta}} - (\partial\alpha_1/\partial\hat{\boldsymbol{\theta}})\dot{\hat{\boldsymbol{\theta}}} - (\partial\alpha_1/\partial\boldsymbol{\xi}_0)\mathbf{n}],\end{aligned} \quad (10)$$

where $\overline{\boldsymbol{\varpi}} = (\partial\alpha_1/\partial y)\boldsymbol{\omega} + [0,0,-z_1]^T$. Then the choices

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\tau}_1 - \boldsymbol{\Gamma}\overline{\boldsymbol{\varpi}}z_2\,, \quad \mathbf{n} = \boldsymbol{\zeta}_1 - d_1\mathbf{P}_0^{-1}(\partial\alpha_1/\partial y)z_2\mathbf{e}_2\,, \quad u = [-c_2z_2 - \beta_2 - \overline{\boldsymbol{\varpi}}^T\hat{\boldsymbol{\theta}} + (\partial\alpha_1/\partial\hat{\boldsymbol{\theta}})\dot{\hat{\boldsymbol{\theta}}} + (\partial\alpha_1/\partial\boldsymbol{\xi}_0)\mathbf{n}]/\sigma(y)\,,$$

with $c_2 > 0$, yield the negative semi-definite derivative $\dot{V}_2 = -c_1z_1^2 - c_2z_2^2 - d_1^{-1}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}$. According to the La-Salle-Yoshizawa convergence theorem [3] for non-autonomous nonlinear systems, the tracking and the observer errors will converge to zero, but in general the parameter estimates will not converge to their true values. All the system signals will be bounded and the reference trajectory will be the only asymptotically stable motion of the closed-loop adaptive system. This implies that the output of the objective nonlinear system will track asymptotically the desired trajectory, which allows the performance specifications of the closed-loop adaptive system to be set with the reference trajectory.

3. ADAPTIVE TRAJECTORY TRACKING CONTROL SYSTEM DESIGN

The mechanical part of the single link manipulator dynamics is described by the equations

$$\dot{\eta}_1 = \eta_2\,, \ \dot{\eta}_2 = b_0u_c - a_1\eta_2 - a_2\sin\eta_1\,, \quad (11)$$

where $\eta_1 = y$ is the angular position, η_2 – the angular speed, a_1, a_2, b_0 are unknown parameters and u_c is the actuator current which produces the driving torque. Via the coordinate transformation $x_1 = \eta_1$, $x_2 = \eta_2 + a_1\eta_1$ the system (11) is transformed into the necessary output feedback form

$$\dot{x}_1 = x_2 - a_1y\,, \ \dot{x}_2 = b_0u_c - a_2\sin y\,, \quad (12)$$

with y as the output. Considering the general description (1) we can specify the vector functions for (12): $\boldsymbol{\varphi}_0 = [\varphi_{01}, \varphi_{02}]^T = [0,0]^T$, $\boldsymbol{\varphi}_1 = [\varphi_{11}, \varphi_{12}]^T = [-y,0]^T$, $\boldsymbol{\varphi}_2 = [\varphi_{21}, \varphi_{22}]^T = [0,-\sin y]^T$ and $\sigma(y) = 1$.

The interlaced adaptive controller-observer method is applied to system (12). The necessary errors are $z_1 = y - y_r$, $z_2 = \upsilon_{02} + \hat{p}\Psi$, with $\Psi = c_1z_1 + \xi_{02} - \dot{y}_r + \hat{a}_1(\xi_{12} - y) + \hat{a}_2\xi_{22}$.

The adaptive control law reads

$$u_c = -c_2z_2 - \beta_2 + \overline{\boldsymbol{\varpi}}^T\hat{\boldsymbol{\theta}} - \hat{p}[(\xi_{12} - y)\dot{\hat{a}}_1 + \xi_{22}\dot{\hat{a}}_2 + \mathbf{e}_2^T\mathbf{n}]\,, \quad (13)$$

where $\overline{\boldsymbol{\varpi}}^T = [-\hat{p}(c_1 - \hat{a}_1)(\xi_{12} - y), -\hat{p}(c_1 - \hat{a}_1)\xi_{22}, -\hat{p}(c_1 - \hat{a}_1)\upsilon_{02} - z_1]$ and

$$\beta_2 = \hat{p}[c_1(\xi_{02} - \dot{y}_r) + k_2(y - \xi_{01}) - \hat{a}_1(\xi_{02} + k_2\xi_{11}) - \hat{a}_2(k_2\xi_{21} + \sin y) - \ddot{y}_r] + \gamma_p\mathrm{sign}(b_0)\Psi^2z_1 - k_2\upsilon_{01}.$$

The dynamics of the estimates, the correction term and the filters are given by the equations

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\{[\xi_{12} - y, \xi_{22}, 0]^T z_1 - \overline{\boldsymbol{\varpi}}z_2\}\,, \ \dot{\hat{p}} = \gamma_p\mathrm{sign}(b_0)z_1\Psi\,, \ \mathbf{n} = d_1\mathbf{P}_0^{-1}\mathbf{e}_2[z_1 + \hat{p}(c_1 - \hat{a}_1)z_2]\,, \quad (14)$$

$$\dot{\boldsymbol{\xi}}_0 = \mathbf{A}_0\boldsymbol{\xi}_0 + \mathbf{k}y + \boldsymbol{\varphi}_0(y) + \mathbf{n}\,, \ \dot{\boldsymbol{\xi}}_1 = \mathbf{A}_0\boldsymbol{\xi}_1 + \boldsymbol{\varphi}_1(y)\,, \ \dot{\boldsymbol{\xi}}_2 = \mathbf{A}_0\boldsymbol{\xi}_2 + \boldsymbol{\varphi}_2(y)\,, \ \dot{\boldsymbol{\upsilon}}_0 = \mathbf{A}_0\boldsymbol{\upsilon}_0 + \mathbf{e}_2u_c\,. \quad (15)$$

The reference input trajectory is generated by the reference model

$$\dot{x}_{1d} = x_{2d}\,, \ \dot{x}_{2d} = -\lambda_2x_{2d} - \lambda_1x_{1d} + \lambda_1v\,, \quad (16)$$

where v is the desired angular position. This model provides the necessary reference variables $y_r = x_{1d}$, $\dot{y}_r = x_{2d}$, $\ddot{y}_r = -\lambda_2 x_{2d} - \lambda_1 x_{1d} + \lambda_1 v$ for the control (13), used as a reference signal for the PI current controller. The manipulator electrical dynamics is described by $\dot{\eta}_3 = -a_3\eta_2 - a_4\eta_3 + a_5 u$, which models a DC motor with η_3 – the armature current, u – the input voltage. The PI controller is

$$\dot{\upsilon}_{03} = (u_c - \eta_3)/T_i,\ \ u = k_p(u_c - \eta_3) + \upsilon_{03}, \tag{17}$$

where T_i is the integration time constant and k_p is the controller gain. The nonlinear adaptive control system consists of adaptive control law (13), estimates dynamics and correction term (14), filters (15), reference model (16) and PI controller (17). Its implementation requires the manipulator angular position and motor current measuring. No angular speed measurement is required which is an advantage.

4. Simulation Results

The adaptive control system designed is simulated and the results are shown in figures 1–6. The

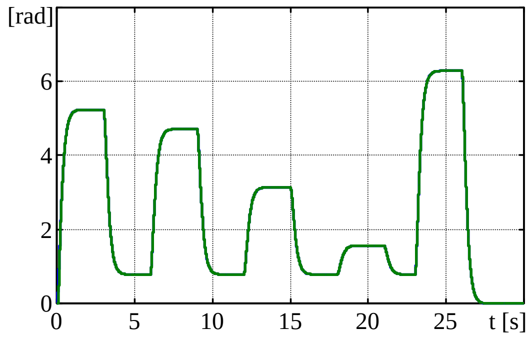

Figure 1: Trajectory tracking

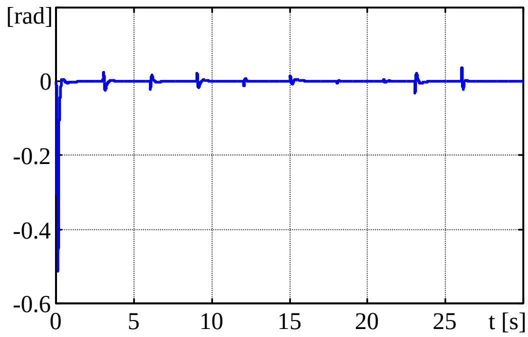

Figure 2: Trajectory tracking error

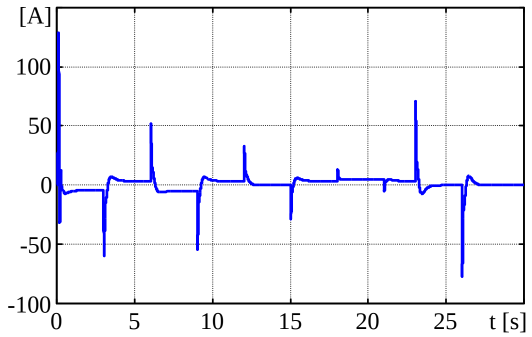

Figure 3: Motor armature current

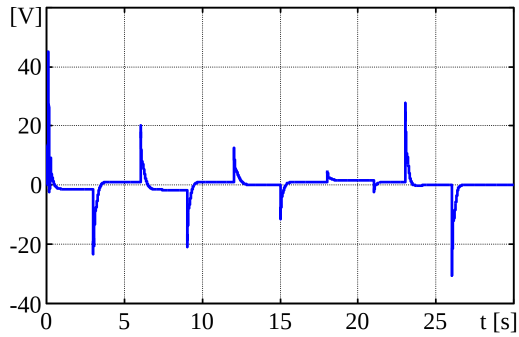

Figure 4: Motor input voltage

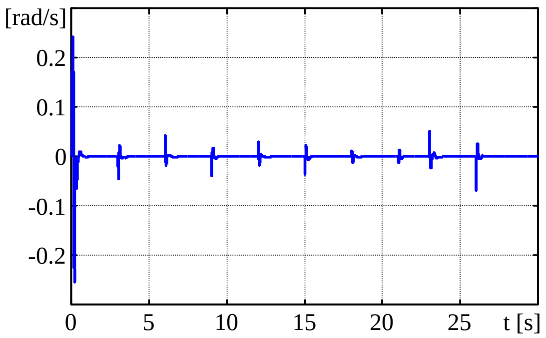

Figure 5: Observer error ε_2

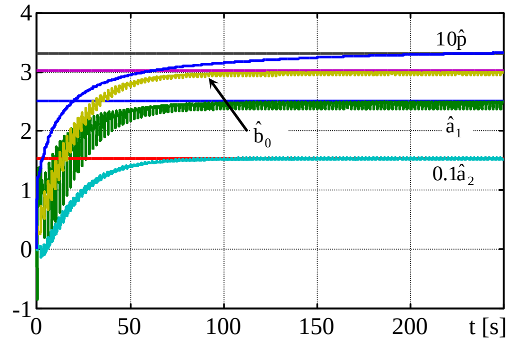

Figure 6: Parameter estimation

proposed control achieves positioning in the desired angles in about a second and a half with an overdamped character of the response, due to the precise tracking of the reference trajectory. The speed of positioning is limited by the motor armature current value. If faster positioning is required more powerful motor has to be used.

5. CONCLUSION

The paper has presented the interlaced adaptive controller-observer approach applied for adaptive nonlinear trajectory tracking control system design of a DC motor driven single link manipulator. The dynamics of the objective system is decomposed into mechanical and electrical parts. The mechanical part is transformed into the nonlinear output feedback form with the motor armature current as input. The derived adaptive control law is used as a reference signal for current tracking PI controller which drives the electrical dynamics of the system. The nonlinear adaptive system is designed to asymptotically track a desired reference trajectory. This leads to an overdamped character of the angular positioning response, which is guaranteed in the entire operating range and is independent of the positioning speed and the desired angle. In this way the closed-loop adaptive nonlinear system performance specifications are precisely controlled with the reference trajectory and can be easily modified. The dynamics of the parameter estimators is designed to be stable but not asymptotically stable, which in general leads to inaccurate estimation. Despite of this, the overall system response is asymptotically stable with respect to the desired reference trajectory according to the design specifications. This property is a consequence of the interlaced adaptive controller-observer design approach because it is based on the Lyapunov stability theory and the nonlinear system model is explicitly taken into account. If linear control is used the tuning of the controller, hence its performance will depend on the plant parameters. Therefore, when these parameters are not exactly known or changing dynamically the linear control performance can deteriorate intolerably. Moreover, the stability of the control system in specific regions or in the entire operating range of the nonlinear system might be lost, which is totally unacceptable. The advantage of the proposed adaptive control is its unique ability to compensate for the completely unknown and possibly changing plant parameters and to guarantee uniform performance in the entire operating range, while maintaining global asymptotic stability of the closed-loop nonlinear system. This guaranteed stability, incorporated into the nonlinear control design, is the reason why the nonlinear control in general is always superior compared to the linear control of nonlinear systems. Furthermore, if the objective system is more complex then high performance control in the entire operating range can only be achieved with nonlinear methods. Another interesting advantage of the proposed adaptive control system is that it tends to improve its performance, in sense of trajectory tracking error and observer error, as the closed-loop control system operates. Additionally, given enough time and a sufficiently rich persistent excitation the unknown parameters can be accurately estimated. However, the exact estimation of the unknown parameters is not part of the adaptive control system design and in fact is not necessary for its high performance operation, as it’s evident from the simulation results given.

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