

# LABORATORY MODEL FOR VERIFICATION OF THE CONTROLLERS

**Michal Dobias**

Doctoral Degree Programme (3), FEEC BUT

E-mail: xdobia04@stud.feec.vutbr.cz

Supervised by: Petr Pivoňka

E-mail: pivonka@feec.vutbr.cz

**Abstract:** This paper deals with the laboratory models whose core is the microprocessor. There are currently three models: DC motor, Water tank and Pool heating. In this article these three models are introduced and verified. The summarizing publication of this area is [1]. This work selects appropriate models and numerical methods which are implemented in the PLC.

**Keywords:** numerical methods, discrete approximation, laboratory models, PI controller, LQ controller

## 1. INTRODUCTION

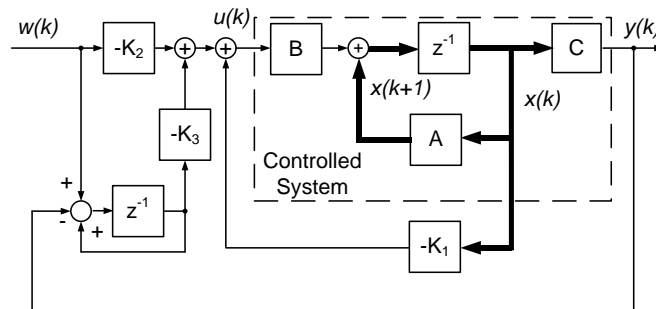
In the eighties of 20th century there has been significant progress in the development of computer technology and the applications of microprocessors has reached a wide range of use. New methods of programming allowed more efficient and quicker development of applications. The most common applications are personal computers, cell phones, in the industry there are the PLCs, Servo drives, HMI, etc. One of the atypical applications is the microchip as a laboratory refund for the real physical system, which can have unsuitable properties for the laboratory use (size, energy consumption, malicious content, and long response times).

## 2. CONTROLLER

PI controller, or more precisely, the discrete equivalent [2] is the most commonly used controller in practice. It is popular for its simplicity, the possibility of intuitive setup of parameters, and also for the wide range of possible method setup.

$$F_R(z) = K \left[ 1 + \frac{Tz^{-1}}{T_I(1-z^{-1})} \right] \quad (1)$$

Relay controller is one of the oldest methods of regulation. It is especially easy to control pressure and temperature. It is an inexpensive alternative of control. These controllers are robust to system parameters changes and are capable of controlling nonlinear systems.



**Figure 1:** Modification of LQ controller.

Linear quadratic optimal controller (LQ) is in its basic form state controller with proportional feedback of system status. The matrix obtained is then optimally chosen to the defined criterion [3] Controller is modified for reference tracking and zero control error (Qse penalizing sum of control error).

$$J = \frac{1}{2} x^T Q x + u^T R u \quad (2)$$

### 3. MODELS

DC motor, Water tank and Pool heating were chosen as the models of real systems. Their mathematical descriptions were discretized and by the use of a numerical method implemented as discrete models.

#### 3.1. DC MOTOR

We use a DC motor with separate excitation. The equations describing its function are:

$$u_m = R_m i_m + L_m \frac{di_m}{dt} + k\omega \quad (3)$$

And for radial speed

$$J \frac{d\omega}{dt} = M_m - M_z = k i_m - M_z \quad (4)$$

$u_m$  ... power supply voltage,  $R_m$  ... winding resistance,  $i_m$  ... winding current,  $L_m$  ... winding inductance,  $k$  ... motor constant,  $\omega$  ... radial speed,  $J$  ... moment of inertia,  $M_m$  ... motor torque and  $M_z$  ... load torque.

Euler's method is used for the numerical solution with the Taylor's expansion in this form [4]:

$$y_{t+1} = y_t + f(x_t, y_t) \cdot h + \frac{f'(x_t, y_t)}{2} \cdot h^2 \quad (5)$$

$$i_m(t+1) = i_m(t) + \frac{u_m(t) - R_m i_m(t) - k\omega(t)}{L} \Delta t + \frac{R_m/L}{2} \Delta t^2 \quad (6)$$

$$\omega_m(t+1) = \omega_m(t) + \frac{k i_m(t) - M_z}{J} \Delta t + \frac{k/J}{2} \Delta t^2 \quad (7)$$

#### 3.2. WATER TANK

The model of Water tank is described by this differential equation:

$$A \frac{dh}{dt} = w - k a \sqrt{2gh} \quad (8)$$

$A$  ... surface of level,  $w$  ... amount of inflowing water,  $k$  ... hydrodynamic factor,  $a$  ... cross section of the outflow,  $g$  ... gravity acceleration and  $h$  ... level height.

The model contains nonlinearity which is the reason why we use a relay controller [5]. Numerical solution is achieved by using standard Euler's method [6]:

$$y_{t+1} = y_t + f(x_t, y_t) \cdot h \quad (9)$$

$$h(t+1) = h(t) + \frac{w(t) - ka\sqrt{2gh(t)}}{A} \Delta t \quad (10)$$

### 3.3. POOL HEATING

For Pool heating model has not been proposed analytical solution because of overall complexity of the issue. Heating pool model uses the knowledge that to heat the pool water by  $1^\circ\text{C}$  is necessary

to supply energy for size: 
$$Q_c = C_t m \left[ J; \frac{J}{kg \cdot K}, kg \right] \quad (11)$$

Sun supplies energy directly by this form

$$Q_s = S_b \eta_b \cdot 3600 \cdot SZ \left[ J; m^2, \%, -, kWh \right] \quad (12)$$

Electric heating supplies energy directly by this form

$$Q_e = P \cdot 3600 \cdot t \left[ J; W, -, h \right] \quad (13)$$

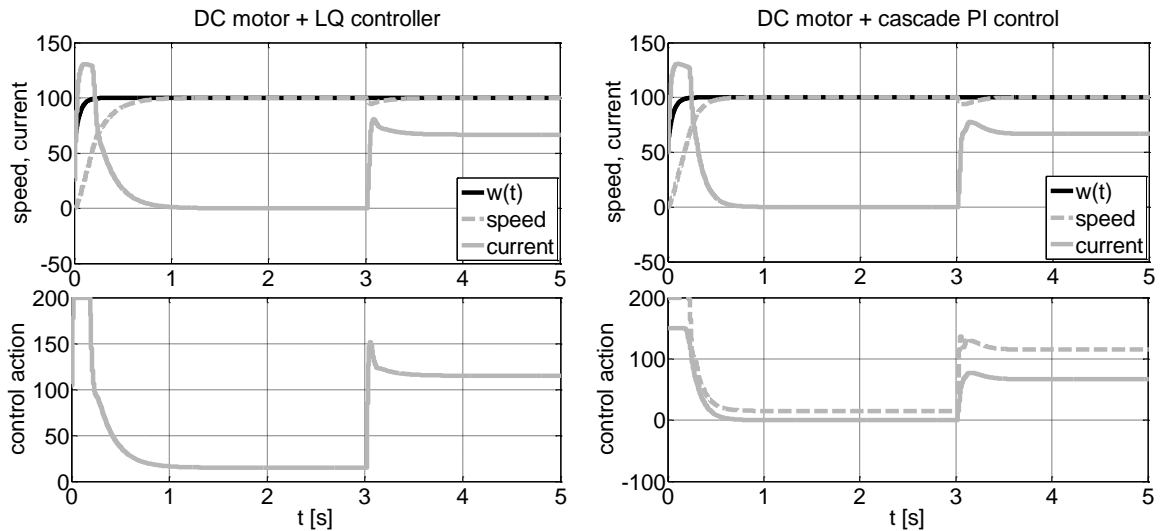
Solar panels supplies energy according to the equation

$$Q_p = S_p \eta_p \cdot 3600 \cdot SZ \left[ J; m^2, \%, -, kWh \right] \quad (14)$$

$C_t$  ... specific thermal capacity,  $m$  ... weight,  $S_b$  ... area of the swimming pool,  $S_p$  ... Solar panel area,  $\eta_b$  ... efficiency of the heat exchange,  $\eta_p$  ... efficiency of the heat exchange,  $SZ$  ... energy of the solar radiation and  $P$  ... power of the electric heater.

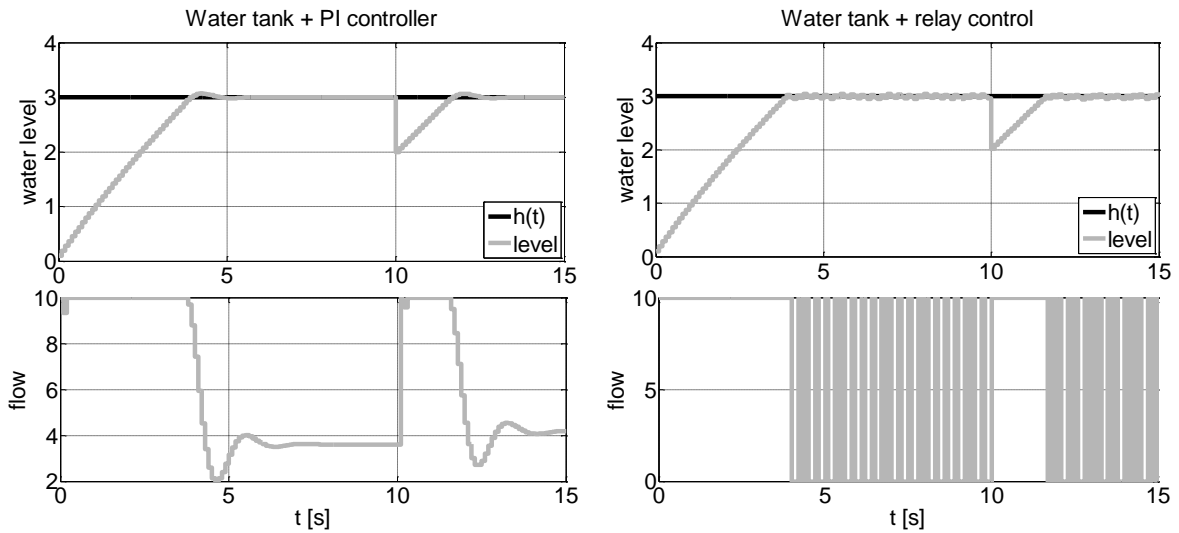
The  $Q_s$ ,  $Q_e$  a  $Q_p$  energy supplied the heating of the pool water. In the swimming pool are the heat losses, which drain the thermal energy. Thermal losses represent  $Q_{odp}$  – evaporation losses, which represent approximately 56 % of total losses. It is also necessary to consider heat exchange between the pool and its surroundings, which depends on the temperature difference between water and air temperature and the wind speed.

## 4. SIMULATIONS



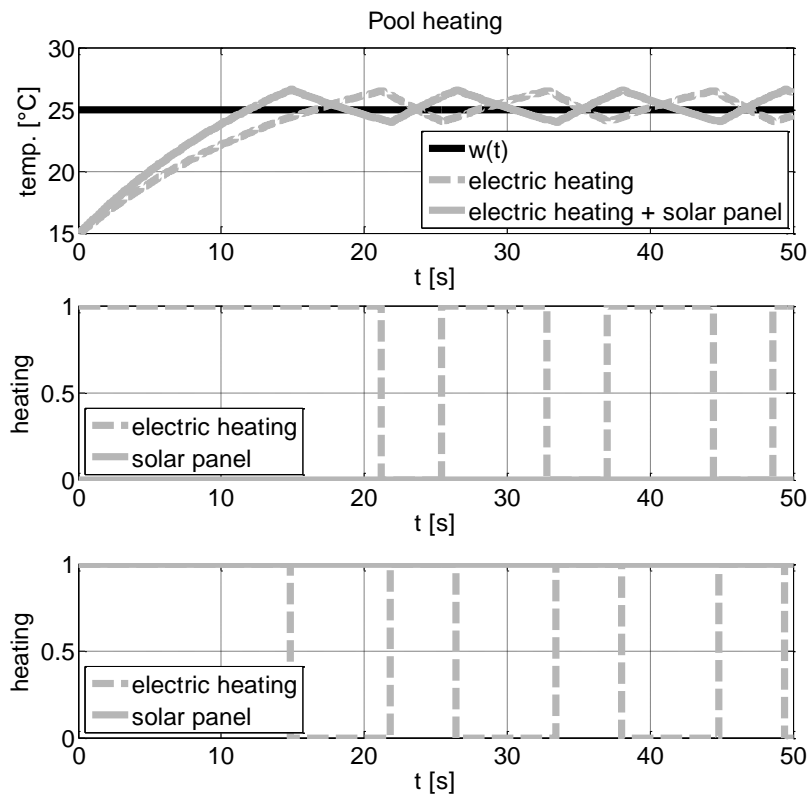
**Figure 2:** DC motor control process (LQ controller:  $Q = 1$ ,  $Q_{se} = 0.003$ ,  $R = 0.0005$ ; cascade PI control: current controller  $K = 3.5$ ,  $T_1 = 0.4$ ; speed controller  $K = 9$ ,  $T_1 = 2$ ).

In Fig. 2 (left) we can see the DC motor control process which is controlled by modified LQ controller. In the right side is the cascade PI control of the DC motor.



**Figure 3:** Water tank control process (PI controller:  $K = 40$ ,  $T_1 = 0.3$ ; relay control: switching area  $(-0.02; 0.02)$ ).

Fig. 3 depicts the Water tank control. In the left side we can see the PI level control. The right side of Fig. 3 presents typical relay control which is used in many industrial applications.



**Figure 4:** Relay pool heating control (switching area  $(1; -1.5)$ ).

In Fig. 4 we can see the heating pool process. There is the comparison between the heating with solar panel and heating without the solar panel.

All simulations, computations and results are obtained with MATLAB-Simulink and B&R Automation Studio. The communication was realized by especially developed library, which connects the B&R PLC with MATLAB-Simulink.

## 5. CONCLUSION

The aim of this work was to design and implement algorithms for laboratory models whose core is the microprocessor. Presented algorithms have been tested for the step response, and also for the step disturbance on the output system (the disturbance is active at the time 3 s for DC motor and 10 s for Water tank). Presented algorithms have satisfactory behavior. Simulation for DC motor runs with sampling time 0.01 s, the Water tank has the sampling period 0.1 s and for the Pool heating was the sampling time 1 s. The advantage of this solution is the possibility of acceleration of the simulation for systems with long time constants. The advantage of modification and of the actual solution is:

- Smaller space requirements, high reliability and lifetime of the compact CPU.
- Simple extension by next models and supply of computing power.
- Simple modifications of the models and maintenance, which can be done without special training.

Possible extensions:

- Heat model of the DC motor.
- Remote access via VNC client for teacher, for activity inspect or remote modification of the model parameters.
- Expanding the number of individual models and their variants (especially DC motors).

## ACKNOWLEDGEMENT

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