WIND TUNNEL IDENTIFICATION AND CONTROL VIA OE MODEL AND LQ CONTROLLER

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ABSTRACT

LQ controller with short sample time was used for control of wind tunnel. Off-line identification via output error model was used to achieve unbiased approximation of the process. States variables were estimated via Kalman's filter and observer realized as 3th order discreet model. Overshot was canceled without lost of stability.

1. INTRODUCTION

In this paper wind tunnel is controlled via LQ controller. Plant is nonlinear and its output is significantly influenced by noise. State space representation is created from input output off-line identification closed to choose operation point. Short sample times T=0.1s is used for both identification and controller. To achieve unbiased approximation of the plant (with significant noisy and short T) output error identification model was used as show figure.1. States of the plant are estimate via Kalman's filter and 3th order discreet model. Control loop properties are compared for both of estimators and the same LQ controller. Simple overshot cancelling is realized without lost of stability.

2. IDENTIFICATION

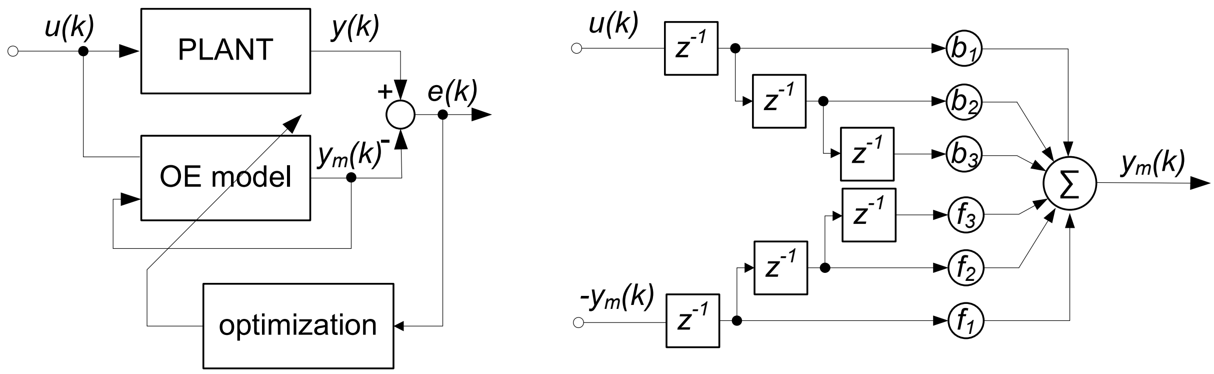

Figure 1: Basic identification scheme and approximate OE model

2.1. OUTPUT ERROR MODEL

OE model is the widely used structure. It is the simplest representative of the output error model structures. The noise is assumed to disturb plant additively at output. Output error models are often more realistic model, and thus they perform better than equation error models. All output error models are nonlinear in their parameters and consequently they are harder to estimate. Optimization algorithm Lavenberg-Marquart will be used for their nonlinearity.

The model of 3 order is used in this case.

$$y_m(k)=\frac{B(q)}{F(q)}u(k) \quad \text{or} \quad F_M(z)=\frac{b_1 z^{-1}+b_2 z^{-2}+b_3 z^{-3}}{1+f_1 z^{-1}+f_2 z^{-2}+f_3 z^{-3}} \tag{1}$$

The model can be rewritten in vector forms as follows

$$\hat{y}(k)=\varphi^T(k)\theta(k) \tag{2}$$

where

$$\varphi(k)=\left[u(k-1)\ u(k-2)\ u(k-3)\ -y_m(k-1)\ -y_m(k-2)\ -y_m(k-3)\right]^T$$

is the vector of measured inputs and outputs and

$$\theta(k)=\left[b_1(k)\ b_2(k)\ b_3(k)\ \ f_1(k)\ \ f_2(k)\ \ f_3(k)\right]^T \tag{4}$$

is vector of estimated system parameters.

2.2. Optimization Algorithm

Identification is based on finding the minimum of cost function

$$I(\theta)=\frac{1}{2}\sum_{k=0}^{n} e^2(k) \quad \approx \quad I(\theta)=\frac{1}{2}E^T E \tag{5}$$

where

$$e(k)=y(k)-y_m(k),\ E=\left[e(0)\ e(1)\ \ \cdots\ \ e(n)\right]^T \tag{6}$$

and Jacobian of *e(k)*

$$\frac{\partial e(k)}{\partial \theta}=-\left[\frac{\partial y_m(k)}{\partial b_1}\ \frac{\partial y_m(k)}{\partial b_2}\ \frac{\partial y_m(k)}{\partial b_3}\ \frac{\partial y_m(k)}{\partial f_1}\ \frac{\partial y_m(k)}{\partial f_2}\ \frac{\partial y_m(k)}{\partial f_3}\right] \tag{7}$$

then

$$J=\left[\frac{\partial e(0)^T}{\partial \theta} \quad \cdots \quad \frac{\partial e(n)^T}{\partial \theta}\right]^T \tag{8}$$

and partial derivation of y_m are

$$\frac{\partial y_m(k)}{\partial b_i}=\frac{1}{F(q)}u(k-i) \tag{9}$$

$$\frac{\partial y_m(k)}{\partial f_i}=-\frac{1}{F(q)}y(k-i) \tag{10}$$

2.3. LEVENBERG-MARQUART ALGORITHM

This method is numerical solution of minimization sum of squares generally nonlinear function. Iterative algorithm is given by equation

$$\theta(i) = \theta(i-1) - \eta\left[J^T J + \lambda I\right]^{-1} JE \tag{11}$$

Where i is iteration in step, η is step size, λ is optional parameter which determine evolution of prediction error and E is error vector.

3. CONTROL ALGORITHM

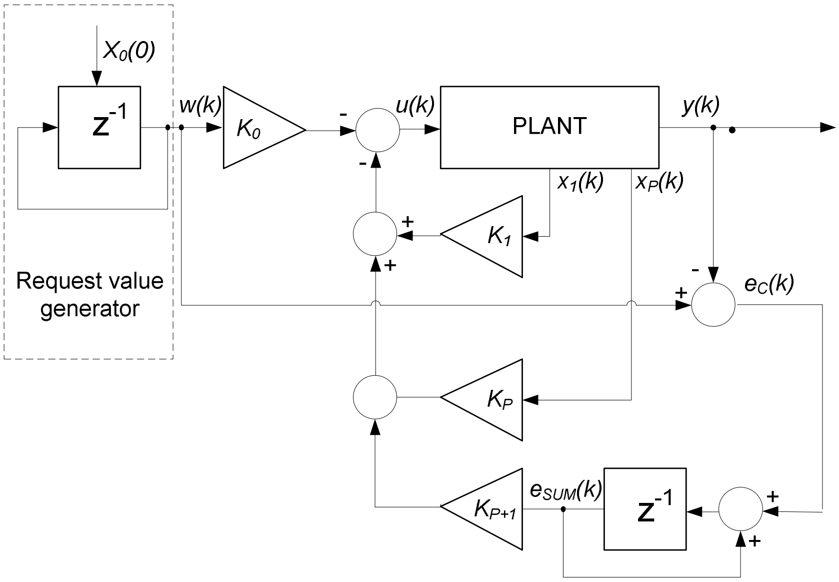

Figure 2: Optimized structure of LQ controller

Linear quadratic (LQ) controller is used. LQ controller is state controller with feedback proportional gains from process states. Optimised cost function is

$$I = \sum_{k=0}^{\infty} \left(e_c^{\,T}(k) Q e_c(k) + u^T(k) R u(k)\right) \tag{12}$$

Where matrix R scale controls energy and matrix Q scales error of system states. Action value is computed by

$$u(k) = -K_{LQ} x(k) = -[K_0 \; K_1 \cdots K_P \; K_{P+1}][w(k) \; x_1(k) \cdots x_P(k) \; e_{SUM}(k)]^T \tag{13}$$

Steady state gain K_{LQ} vector is solved as iterative compute of follow

$$\begin{aligned} K_{LQ} &= \left[R + B^T P_{LQ} B\right]^{-1} B^T P_{LQ} A \\ P_{LQ} &= Q + K_{LQ}^T R K_{LQ} + \left(A - BK_{LQ}\right)^T P_{LQ}\left(A - BK_{LQ}\right) \end{aligned} \tag{14}$$

Where K_{LQ} are state feedback gain, $w(k)$ is request value, A and B are state space matrix of $e_c(k)$ without controller.

3.1. OBSERVER

Because the plant states aren't measurable, state observer must be used. Two observers will be used. Kalman's filter and observer based on 3th order linear discreet model of plant. State space realization is getting from off-line identification. OE model identification ensure that obtained model will be very good approximation of identificated plant.

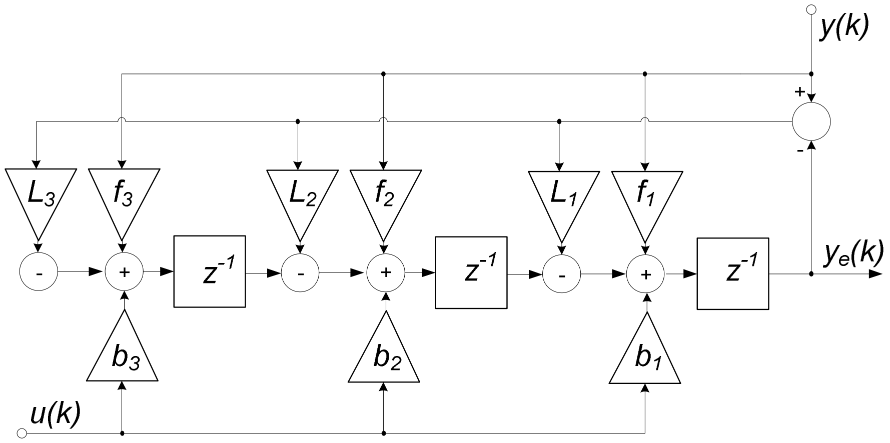

Figure 3: State space representation of Kalman's filter with Kalman gain L, 3th order linear discreet observer for L=0

Kalman filter gain L is computed as steady state solution of equation (15) below. For 3th order linear discreet observer gain L is set to zero.

$$
\begin{aligned}
P &= \alpha^2 APA' + Q \\
L &= PC(C^T PC + R)^{-1} \\
P &= (I-LC^T)P(I-LC^T)' + LRL',
\end{aligned}
\tag{15}
$$

Where A is system matrix of plant, covariance matrix Q and R are used for model error compensation, α is 1, P covariance matrix.

4. RESULTS

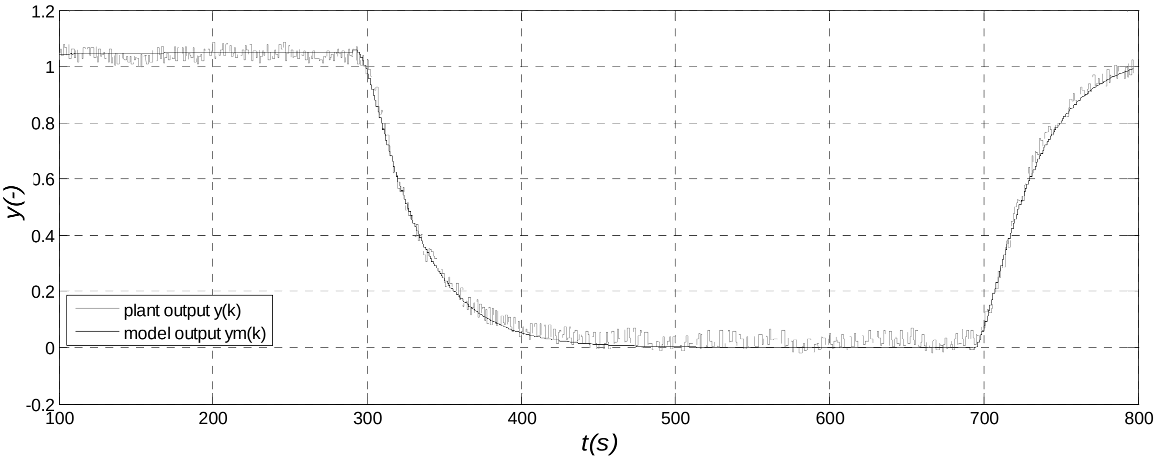

Figure 4: Off-line identification results, model and plant response to the square input signal, *Ts=0.1s*

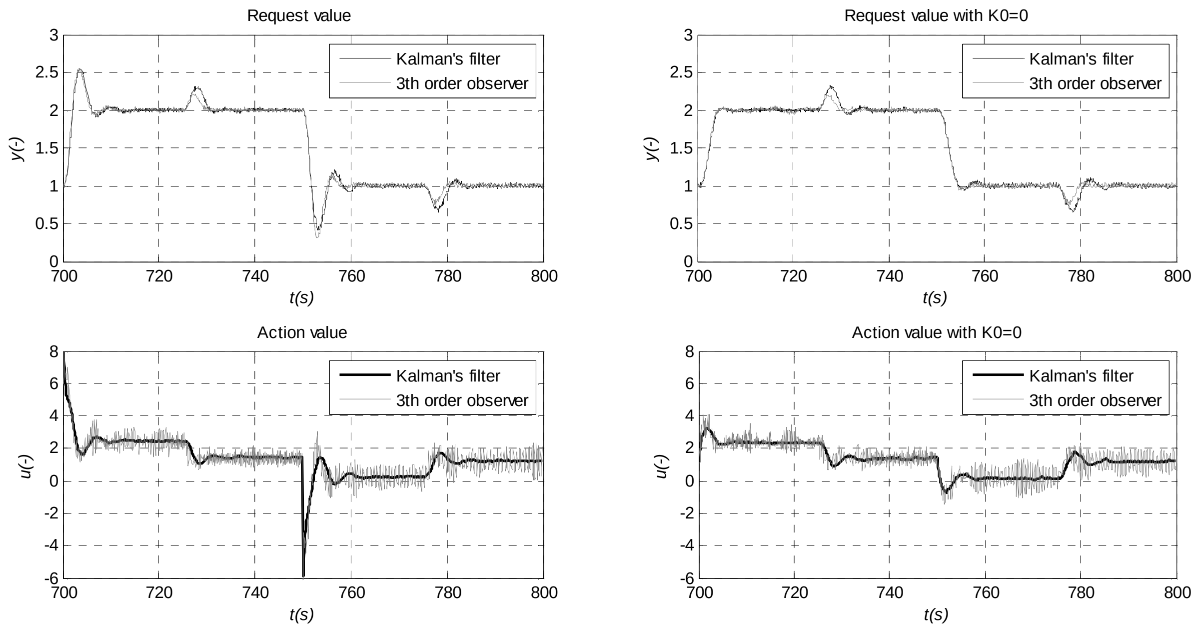

Figure 5: Graphs of controlled value and action value with the same LQ controller the left, identical controller with changed $K_0=0$ right, $T_S=0.1s$, disturbance between *725s* and *775s*.

5. CONCLUSION

Wind tunnel was controlled with regulator, whose structure is shown on Figure 2. Structure of regulator is optimized with LQ algorithm according to criteria (12). Identification based on OE model was used to obtain system description, the similarity rate show Figure 4. State estimation was implemented with Kalman's filter and with observer based on 3th order linear discreet model. Processes of values in closed loop with both types of observers can by seen in Figure 5. Control loop with 3th order linear discreet observer (deadbeat observer) achieved better results only for disturbance cancelling but action values are not absolutely suitable for practical application. Figure 5 right presents overshoot cancelling, which is achieved via forward gain K_0.

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