A PRIORI CONDITIONS FOR EFFICIENT IMAGE RECONSTRUCTIONS

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ABSTRACT

The image reconstruction problem based on electrical impedance tomography is still a widely investigated problem with many applications in physical and biological sciences. The paper proposes a new way to improve the quality of the image reconstruction of electrical impedance tomography. A developed algorithm the advantages of the Tikhonov regularization method with advantages of the Level set method. The significant improvement is also reached when a priori conditions are introduced. In this paper the obtained reconstruction results are presented

1. INTRODUCTION

Electrical Impedance Tomography (EIT) belongs to methods, which can be used for its good detection of conductivities tissue changes. To find the distribution of unknown conductivity inside the investigated object we can use methods, which are based on the deterministic or stochastic approach. In the stochastic approach, only the absolute conductivity in each element is computed and a picture of different conductivity is imaged. In the deterministic approach, temporal variations in conductivity are computed. Individually they can not provide stable and accurate solutions; therefore here we propose a new algorithm.

2. INVERSE PROBLEM

The inverse problem of EIT can be described as an image reconstruction. The aim is to find the unknown conductivity distribution inside the investigated object. In mathematical terms the inverse problem can be represented as a minimization of the suitable objective function $\Psi(\boldsymbol{\sigma})$ of $\boldsymbol{\sigma}$. To minimize the function $\Psi(\boldsymbol{\sigma})$ a deterministic approach based on the Least Squares method [1], [2] can be used.

The back image reconstruction is highly ill-posed inverse problem and so it is necessary to use some regularization. There are different methods of regularizations. One of them is the Tikhonov Regularization method (TRM) [3]

$$\min_{\sigma} \Psi(\sigma) = \min_{\sigma}\left[\frac{1}{2}\sum\left\|U_M - U_{FEM}(\sigma)\right\|^2 + \alpha\left\|L\sigma\right\|^2\right]. \tag{1}$$

Here $\boldsymbol{\sigma}$ is the volume conductivity distribution vector in the object, U_M is the vector of measured voltages on the boundary, and $U_{FEM}(\boldsymbol{\sigma})$ is the vector of computed peripheral voltages relatively to $\boldsymbol{\sigma}$, which can be obtained using FEM, α is a regularization parameter and L is a regularization matrix. To find the solution of (1) we applied the Newton-Raphson method and after the linearization we used the iteration procedure

$$\sigma_{i+1} = \sigma_i + (J_i^T J_i + \alpha L^T L)^{-1} (J_i^T (U_M - U_{FEM}(\sigma_i)) - \alpha L^T L \sigma_i) . \qquad (2)$$

Here index i is the i-th iteration and J is the Jacobian index for the forward operator U_{FEM}. The advantages of this method include the fast convergence and good reconstruction quality. The disadvantages include the possibility to be trapped in local minima and therefore a stable solution is not provided. It is necessary to mention that the stability of the TRM algorithm is also sensitive to the setting of correct initial values of unknown conductivity. The influence on the reconstructions stability has also the number of unknown values, so called Degrees of Freedom (DOF's), and an optimal choice of the regularization parameter α.

Another method is the Level Set method (LSM) [4 – 6], which is designed to identify regions with different image or material properties. The distribution of unknown conductivity $\boldsymbol{\sigma}$ can be described in terms of the level set function F depending on the position of the point $\boldsymbol{r}$ with respect to the boundary Γ between regions with different values of $\boldsymbol{\sigma}$

$$\sigma(r) = \begin{cases} \sigma_{\text{int}} \ \{r : F(r) < 0\} \\ \sigma_{\text{exp}} \{r : F(r) > 0\} \end{cases} , \quad \Gamma = \{r : F(r) = 0\}. \qquad (3)$$

Then the final conductivity distribution $\boldsymbol{\sigma}(\boldsymbol{r})$ represents the steady state for sufficiently large time t of the following time-dependent Hamilton-Jacobi equation

$$\frac{\partial \sigma}{\partial t} + F \left| grad \sigma \right| = 0 , \quad t \to \infty . \qquad (4)$$

The combination of these two methods (TRM and LSM) gives the possibility to obtain a new algorithm which will improve the stability and accuracy of EIT reconstructed images. During this iteration process, which is based on minimizing of the objective function $\boldsymbol{\Psi}(\boldsymbol{\sigma})$, the boundary Γ is searched in accordance with the request that the $\boldsymbol{\sigma}(\boldsymbol{r})$ minimizes the $\boldsymbol{\Psi}(\boldsymbol{\sigma})$, too. We suppose that the unknown conductivity distribution is given by a piecewise constant function $\boldsymbol{\sigma}(NE)$.

The new algorithm can be described generally as follows:

- set constraints for conductivity values;
- initialize $\boldsymbol{\sigma}(NE) = \boldsymbol{\sigma_0}(NE)$, set parameter α, set $\Psi_0(\boldsymbol{\sigma}) = \Psi(\boldsymbol{\sigma_0})$;
- **IF** $\boldsymbol{\Psi}(\boldsymbol{\sigma})$ is decreasing, **THEN**
  - run iteration procedure based on TRM in accordance with (2);
  - set interfaces between subregions with different conductivities in accordance with (3), (4);
  - reduce the number of elements with unknown conductivity values considering the constraints,
- **IF** $\boldsymbol{\Psi}(\boldsymbol{\sigma})$ is still decreasing, **THEN**
  - run new iteration in accordance with (2) and with interactive updating of interfaces.

3. EXAMPLES AND RESULTS

For a new algorithm we will use the objects with biological tissues. The model of 2D arrangement with the original conductivity distribution (Fig. 1) is used to simulate voltage U_M. The conductivity values of different biological tissues are presented in Tab. 1; these values were taken from previously published literature on this theme. The total number of FEM mesh elements is NE = 300.

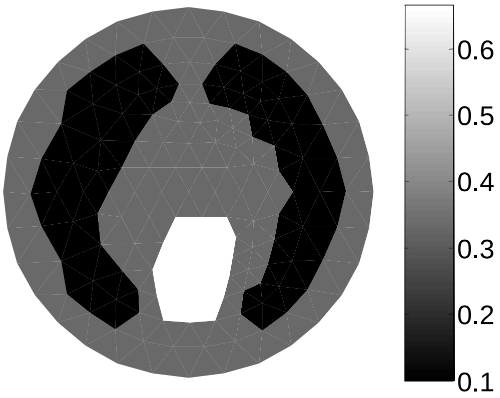

Figure 1: The 2D model for simulation

Region name	Region color	Conductivity σ [S/m]
Homogenous region	Gray color	0.333
Heart	White color	0.667
Lungs	Black color	0.100

Table 1: The conductivity values of biological tissues.

The new method was tested under different conditions. In the first case we identified non-homogenous regions and we supposed that the conductivity values were unknown. The reconstruction results are shown in Fig 2; on the left you can see the result after using TRM and LSM. The final reconstruction results obtained after applying the second TRM are shown in Fig. 2 on the right.

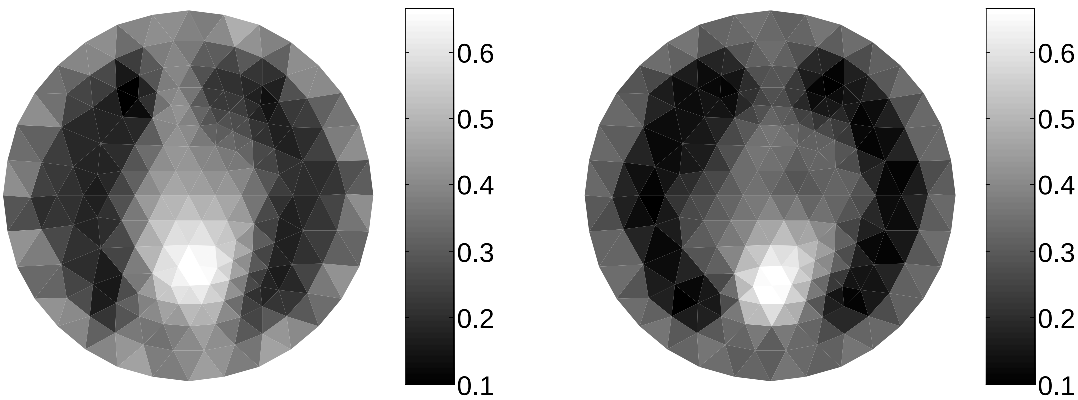

Figure 2: Reconstruction results after using TRM and LSM (left), results after the second using TRM (right)

In the second case we identified non-homogenous regions under the stipulation that the values of the conductivity of all components inside the investigated object (tissue, heart and lungs) were known. The conductivity distribution after using TRM and LSM is shown in Fig. 3 on the left. This figure is result of the reconstruction process, when the default values of $\boldsymbol{\sigma_0}$ for the iteration process can be arbitrarily chosen (positive values). In the same figure on the right we can see the result after the second applying of TRM. In this case the identical distribution as in the original (Fig. 1) was obtained. The new algorithm provides better results, because the image reconstruction passes to simpler case.

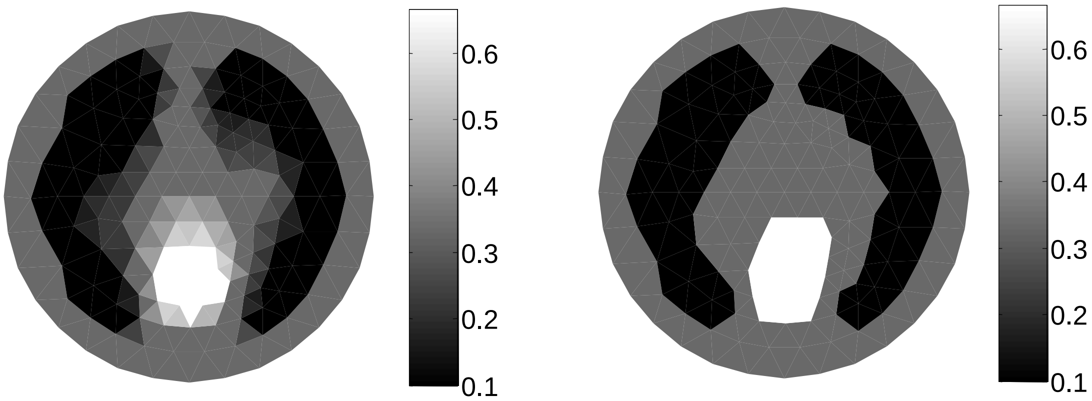

Figure 3: Reconstruction results after using TRM and LSM (left), results after the second using TRM (right)

The advantages of an improved algorithm are shown for these two above described examples in the Tab. 2. In this table the number of DOF's and values of the objective function $\Psi(\boldsymbol{\sigma})$ are compared with TRM and with the new method based on TRM and LSM with a priori conditions.

Step of iteration		Number of DOF's		Objective function $\Psi(\boldsymbol{\sigma})$	
TRM	TRM+LSM	TRM	TRM+LSM	TRM	TRM+LSM
0	0	300	300	1.0e+05	1.0e+05
15	68	70	15	1.1e+03	2.7e-21

Table 2: Comparison of DOF's, $\Psi(\boldsymbol{\sigma})$ for TRM and TRM+LSM during reconstructions.

4. CONCLUSION

In the paper the idea of the new algorithm of the EIT image reconstruction is presented. The new algorithm combines advantages of the Level Set method and Tikhonov regularization method together with a priori conditions applying. Based on lots of different tests it is possible to say, that this new way especially with using of a priori conditions offers a stable and useful tool for an efficient image reconstruction of biologic tissues and their changes.

REFERENCES

[1] Cheney, M., Isaacson, D., Newell, J. C.: Electrical impedance tomography, SIAM Rev., vol. 41, No. 1, 1999, s. 85-101.

[2] Holder, D. S.: Electrical impedance Tomography, IOP Publishing, Philadelphia, 2005.

[3] Vauhkonen, M., Vadász, D., Karjalainen, P. A., Somersalo, E., Kaipio J. P.: Tikhonov regularization and prior information in electrical impedance tomography. IEEE Trans. Med. Eng., Vol. 17, 1998, s. 285–293.

[4] Burger, M. A level set method for inverse problems, Inverse Problems 17, 2001, s. 1327-1356.

[5] Sethiah, J. A.: Level set methods and fast marching methods. Cambridge, Cambridge University Press, 1999.

[6] Osher, S., Fedkiw, R.: Level set methods and dynamic implicit surfaces, Springer-Verlag, New York, 2002.