# **FRICTION MODELS IN SERVODRIVES**

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#### ABSTRACT

This thesis deals with friction in servodrive application. There are described principles of friction generation. With help of some friction models there are described its basic behaviour. In program Matlab/Simulink is simulated an influence of these models on servodrive. It is also shown possibility of approximation mathematical friction model applying neural network.

## **1. INTRODUCTION**

A physically process of friction occurs in all mechanical system. Friction is complex functional of many parameters and there is still no compact theory of it. Because friction directly influences behaviour of mechanical system, it is suitable to know its character and influence by controlling a system. By forgetting it, it can cause some deviations, which devalue the whole system. Friction influences us system controlling with its friction force (torque). From the point of view of controlling, friction effects as torque disturbance. It is necessary to know character of the friction force (torque) to compensate its influences. We can simulate friction static and dynamic behaviour with help of the mathematical models. These models approximate then the real friction. Accuracy of approximation is mostly given by complexity of the model. One of many possibilities is approximation by applying neural network. Advantage of this method is no need to exactly know subject of approximation. So we don't know by friction in deed too.

# 2. CLASSICAL STATIC FRICTION MODELS

Fiction is the tangential reaction force between two surfaces in contact. Physically these reaction force is the result of many different mechanisms, which depend on contact geometry and topology, properties of volume and surface materials of bodies, displacement a relative velocity of bodies, presence of lubrications, pollutions and so on. The main part of static friction is so-called Coulomb friction. By pressing surfaces of two bodies, a lot of micro contacts are created. This is caused by roughness on surface of the bodies. Number of these contacts depends on the normal force  $F_N$  and elastic and plastic deformation, thus on hardness of the materials H. By high pressure the surfaces became closer and the molecules are reacting just by the coulomb forces. Other part of the friction force is so-

called static friction. The force required to overcome the static friction and initiate motion is called Break-away force. Maximal Break-away force depends on surface of the material and on the time when there is no motion. It was find out experimentally that the breakaway force decreases by increasing external force. A big importance for the friction decreasing has a using of lubrication. New mechanisms of the friction appear by adding lubrications. Their principles are based on hydro-dynamical features and parameters of the appropriate lubrication (viscosity, pollution, temperature influences, wearing, and so on.). By using lubrication the friction force mainly depends on relative velocity of the surfaces in contact.

### **2.1.** COULOMB FRICTION MODEL

This model is based on the Coulomb friction and the friction force can be described as:

$$F_t = \overline{\gamma}_C \cdot \operatorname{sgn}(v) \tag{1}$$

where coulomb friction  $F_C = \mu F_N$ ,  $\mu$  is fiction coefficient depending on hardness of material and  $F_n$  is normal force depending on weight of material. You can see that the friction force is not set exactly around zero velocity. For exactly formulation is necessary to extend the formula (1). This simply coulomb model is often extended by the break-away force and linear viscose friction. You can see the friction force at fig.1



Figure 1: Coulomb model with static fiction and linear viscose friction.

# 2.2. STRIEBECK FRICTION MODEL

This friction model combines classical friction (coulomb, break-away force) and so-called Stribeck friction. Stribeck observed that the friction force does not decrease discontinuously as classical models, but that the velocity dependence is continuous. Simulating of the friction force is based on stirbeck curve, which describes a junction between dry and viscose friction. Thus friction force is described as:

$$F_{t} = \begin{cases} F_{C} + (F_{S} - F_{C})^{-\frac{|v_{S}|}{|v_{S}|}^{s_{S}}} + F_{v} \cdot v & pro \ v \neq 0 \\ F_{t} = \begin{cases} F & pro \ v = 0 \ a \ |F| < \overline{\gamma}_{S} \\ F_{S} \operatorname{sgn}(F) & pro \ v = 0 \ a \ |F| \ge \overline{\gamma}_{S} \end{cases}$$
(2)

where  $v_S$  is so-called stribeck velocity.

# **3. DYNAMIC FRICTION MODEL**

The main feature of the dynamical friction model is a hysteresis. The friction force is lower for decreasing than for increasing velocities. The hysteresis loop becomes wider as the velocity variations become faster.

#### **3.1. DAHL FRICTION MODEL**

The Dahl model was developed for adaptive friction compensation in servo system with ball bearings. The ball bearings friction is very similar to solid friction (coulomb friction, etc.). The Dahl model is based on stress-strain curve in classical solid mechanics. Dahl modeled the stress-strain curve by a differential equation and modified it into the time domain as:

$$\frac{dF_t}{dt} = \sigma \ v - \frac{\sigma}{F_c} \frac{F_t}{F_c} |v|$$
(3)

You can see relation with classical friction. Maximal value which the friction force can reach is just coulomb friction.

#### **3.2.** LUGRE FRICTION MODEL

The LuGre model as dynamic fiction model is related to the bristle interpretation of friction. Friction is modeled as the average deflection force of elastic springs. When a tangential force is applied, the bristles will deflect like springs. If the deflection is sufficiently large the bristles start slipping. In contrast to Dahl model, this model includes a break-away force, stribeck and viscose friction in additional. For the friction force we can write:

$$F_t = \sigma_z z + \sigma_z \frac{dz}{dt} + f(v) \qquad \frac{dz}{dt} = v - \tau_z \frac{|v|}{g(v)} z \tag{4}$$

where variable z is introducing a small displacement of the bristles. The function g(v) is representing a stribeck friction, see formula (3). The function f(v) is representing viscose friction, it can be nonlinear too. Stiffness coefficient  $\sigma_0$  tell us how fast a friction force increase when the external force is applied. Damping coefficient is  $\sigma_1$ . The Lugre model is often using in servo drive, because it connects a lot of character of friction. Disadvantage of this model is that it does not describe processes before starting motion.



Figure 2: Friction force of the Dahl and LuGre model for pulse input signal.

#### 4. APPROXIMATION LUGRE MODEL APPLYING NEURAL NETWORK

Neural networks are based on rational imitating structure and principles of biological neural networks with help of technical instruments. One of the most important properties of neural networks for control purposes is the universal approximation property.

## 4.1. APPROXIMATION LUGRE MODEL APPLYING NEURAL NETWORK

We use a Lugre mathematical model for approximation. For neural network design we can use experimental dates as well. We can separate the whole design into several steps. In the first step we suggest in which way neural network will be connected to the approximated system and we choose a training method too. It is a relation between these two operations. For approximation is suitable back-propagation method. This method is based on minimizing mean square errors of network and system outputs. For Back-propagation training is suitable serial-parallel connection of network and system. System outputs go into network inputs. It provides a better convergence. To minimize the mean square error was chosen the Levenberg-Marqvard algorithm. It provides fastest convergence for the networks up to the hundred parameters. This algorithm is based on Newton optimization methods. Levenberg-Marqvard algorithm is described as:

$$W(k+) = V(k) - \int_{-}^{T} J + \mu \int_{-}^{T} J^{T} e$$
(3.3.1)

We have to chose an architecture of the connection between network and system, that means how many system inputs and outputs, with some delay, will be set to input of the network. By using outputs and inputs with delay we can improve stability of the network. We have to chosen a number of preceptrons in layers, number of layer and type of performance function. All of this has an influence on accuracy of approximation. Now we have a complete architecture of the network. In the next step we create a training set. It is suitable to choose max. and min. values in the same range as input signal. In our case it means the velocity of the servo drive. If the training signal is very variable in its shape it will be higher probability that the neural network will be better approximate various type of signal. Now we can start to train the network. We have to choose the accuracy of approximation, initial conditions etc. After training we disconnect the network from the system and than we test itself for various input signal. When the results are not sufficient we make changes in previous steps. You can see the response of the neural network for two various velocity signals at figure 3.



Figure 3: Fricton force of the neural network for two different signals.

# 5. CONCLUSION

By design of servo drive with friction we can choose from wide range available models of friction. With choice of complexity and type of model we determine accuracy of real friction as well as real behaviour of the servo drive. Coulomb model is the most used model in servo drive, thanks to its simplicity. The LuGre nonlinear model is often used in systems with ball bearings and cam chain guides. We can use proper features of the neural networks by approximation of the friction.

# ACKNOWLEDGEMENT

This problem was solved with the support of the project MSM 0021630516 "Výzkum zdrojů, akumulace a optimalizace využití energie v podmínkách trvale udržitelného rozvoje".

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