ELEKTROMAGNETIC TRANSIENTS MATHEMATIC MODELING USING THE TRAPEZOIDAL RULE

Branislav Bátora
Doctoral Degree Programme (1), FEEC BUT
E-mail: xbator00@stud.fekt.vutbr.cz

Supervised by: Petr Toman
E-mail: toman@fekt.vutbr.cz

ABSTRACT

This article deals with the issue of district problems mathematic modeling for electromagnetic transients. Paper analyze iterative methods according to trapezoidal rule by means of concrete application on serial oscillatory RLC circuit. Result study is point out accuracy modeling method and so on factors whereupon is this accuracy dependent.

1. INTRODUCTION

More and more topical problems of transients, overvoltages and switching operations bring the need for better and more efficient methods of modelling and simulating such events. In this area, a specific role is played by the trapezoidal rule belonging to an extensive group of iterative methods. The paper deals with the calculation of a given oscillating circuit, showing the precision and easy application of this method.

2. ELECTROMAGNETIC TRANSIENTS IN THE POWER SYSTEM

Electromagnetic transients appear in circuits containing inductivity and capacitance. There is an energy exchange in the circuit where the energy of the electrostatic field of capacitor C changes into the electromagnetic field of inductor L. Energy losses are due to resistance R (conversion to heat according to the Joule-Lenz law) and to the capacitor dielectric. The paper focuses on oscillating behaviour in such circuits. Temporary overvoltages and overcurrents can appear in the power systems and they must be dealt with by protective systems and the design of power system equipment.

3. MATHEMATICAL DESCRIPTION OF THE OSCILLATING CIRCUIT

In a simplified approach, the oscillating circuit is represented by a series RLC circuit (show in figure 1.) described by the following differential equation:

$$\frac{d^2 u_c(t)}{dt^2} + \frac{R}{L}\frac{du_c(t)}{dt} + \frac{1}{LC}u_c(t) = \frac{U_m}{LC}\sin(\omega t + \varphi) \qquad (1)$$

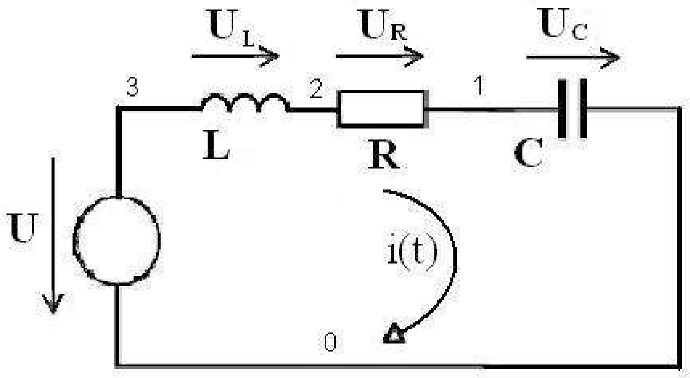


Fig. 1. Series RLC circuit.

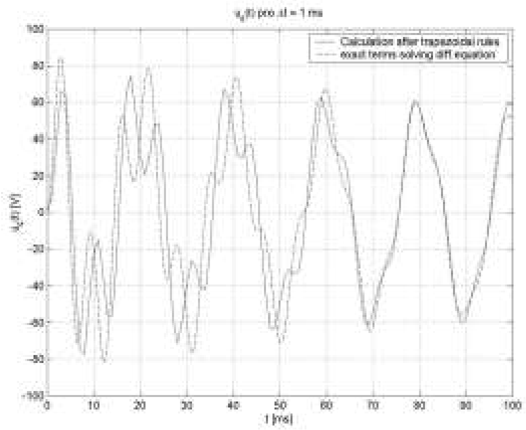
The initial conditions of the transient are:

$$u_c(0) =), \left. \frac{du_c(t)}{dt} \right|_{t=} =) \tag{2}$$

The transformation of the differential equation according to the trapezoid rule [1] and its modification yields the initial set of linear equations:

$$\begin{bmatrix} \frac{2C}{\Delta t} + \frac{\Delta t}{2L} & -\frac{\Delta t}{2L} & 0 \\ -\frac{\Delta t}{2L} & \frac{1}{R} + \frac{\Delta t}{2L} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{1}{R} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_i(t) \end{bmatrix} = \begin{bmatrix} -I_{12}(t - \Delta t) - I_{10}(t - \Delta t) \\ -I_{21}(t - \Delta t) \\ 0 \end{bmatrix} \tag{3}$$

Graphical representation of the voltage waveforms for different values of iterative step shown in Fig. 2 can be obtained by their solution. For comparison, the solution based on the trapezoidal rule (visible full line) is accompanied by the exact solution of the differential equation (visible section line). The calculations were performed in Matlab [2].



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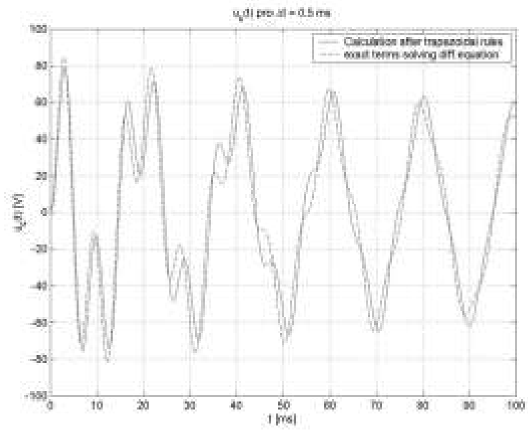

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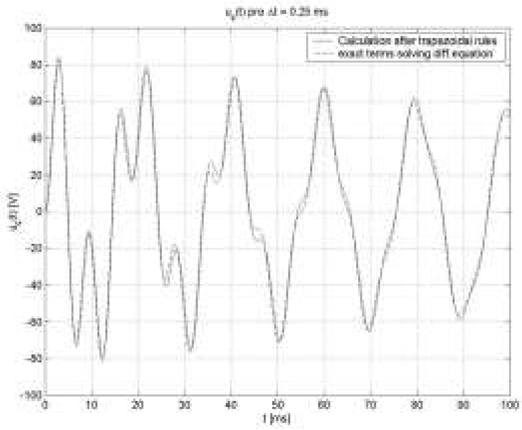

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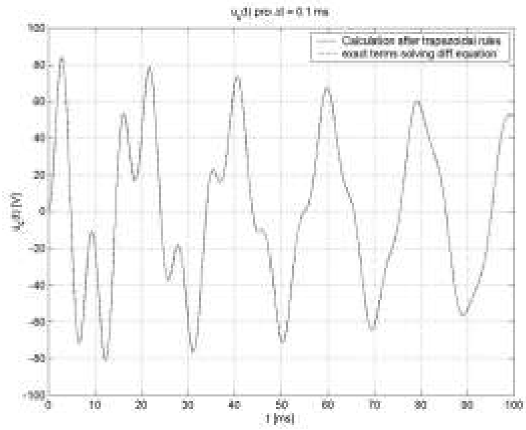
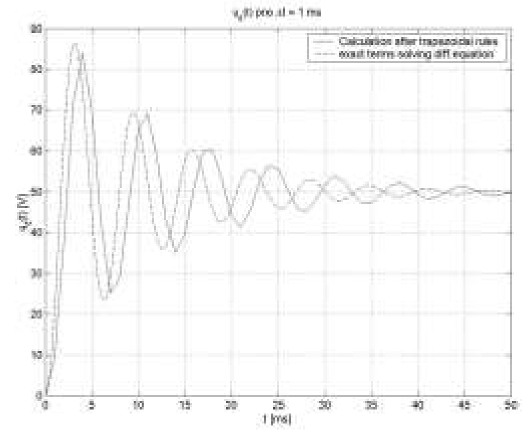


d)

Fig. 2. Waveforms of capacitor voltage in a series RLC circuit supplied by an AC source. a) iterative step 1 ms, b) 0,5 ms c) 0,25 ms d) 0,1ms - resonance period 0,07ms.

It is clear from the presented results that the precision of the method depends on the chosen value of iterative step. The smaller the step, the more precise the result. The most convenient is to choose the iterative step according to the circuit resonance frequency. The result can be considered sufficiently precise if the iterative step is smaller than the inverse value of the circuit resonance frequency.

Fig. 3 shows the behaviour of the oscillating circuit connected to a DC source. The conditions concerning the correct choice of iterative step are same as in case of an AC source.



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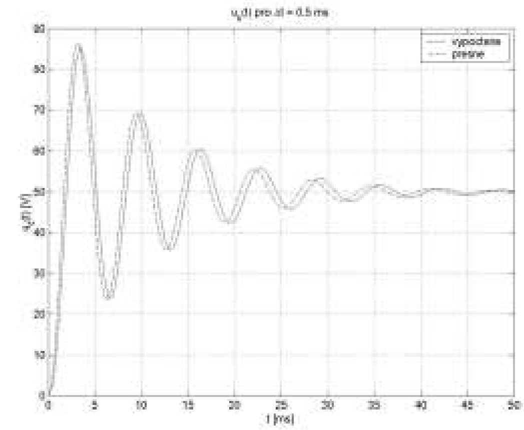

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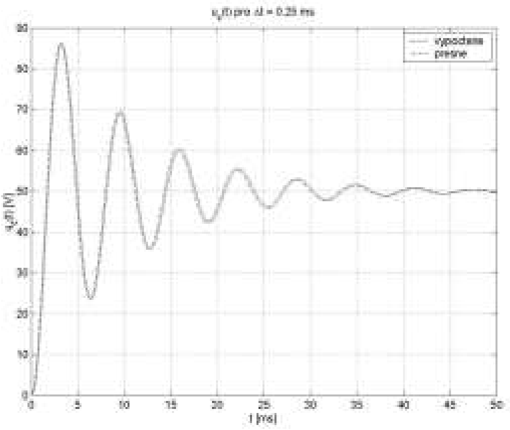

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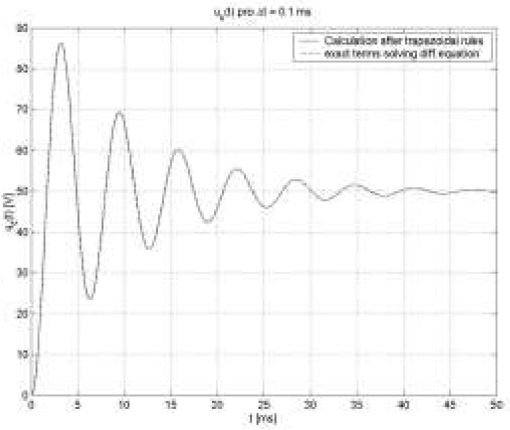

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Fig. 3. Waveforms of capacitor voltage in a series RLC circuit supplied by a DC source. a) iterative step 1ms, b) 0,5 ms, c) 0,25ms, d) 0,1ms.

4. CONCLUSIONS

The application of Fourier and Laplace transformation to computer-based transient analysis is limited because there are mathematical operations that cannot be specified in advance. Most existing software applications for the simulation of elektromagnetic transients in time domain are based on the Bergeron method that uses linear relations between voltages and currents and considers them constant. This assumption does not completely correspond to reality and the steps (iterations) of computer-based solution bring about errors that may lead even to numerical instability. That is why the application of the trapezoidal rule for integrating ordinary differential equations can be very useful.

The method of electromagnetic transient analysis using the trapezoidal rule can be successfully applied to more complicated circuits, obviously with increased requirements on computational capacity and the length of calculation time.

In regard to the fact, that the report deals with the possibility of solving the electromagnetic transient, it´s possible to use the early mentioned observations also for the solving of the problem included in my dissertation work in the area of ferro-resonance overvoltage in the power system.

ACKNOWLEDGMENTS

This paper was made with project MSM0021630516 support of The Ministry of Education, Youth and Sports of The Czech Republic.

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