CALIBRATING ULTRASONIC COMPUTED TOMOGRAPHY SYSTEM USING A MODIFIED GPS ALGORITHM

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ABSTRACT

This paper presents a new method for geometrical self-calibration of an ultrasonic computed tomography (USCT) system. The algorithms used here are based on the so called pseudo-range equations used in the global positioning system (GPS) navigation. Ultrasonic transmitters and receivers in USCT can be viewed as satellite transmitters and mobile receiver units in GPS. However, unlike in GPS, the positions of both transmitters and receivers in USCT are not known. The presented method is capable of calibrating positions and time delays of all ultrasonic transducers at once. No calibration phantoms are necessary.

1. INTRODUCTION

Ultrasonic Computed Tomography (USCT) is a relatively new imaging modality primarily aimed at breast cancer diagnosis. The imaged object is placed in a tank (filled with water as a coupling medium), covered with several thousands of ultrasonic transducers (Figure 1). Each of these transducers is used for either transmitting or receiving ultrasonic pulses. The recorded signals, so called A-scans (Figure 3), can then be used for reconstruction of tomographic images of the object.

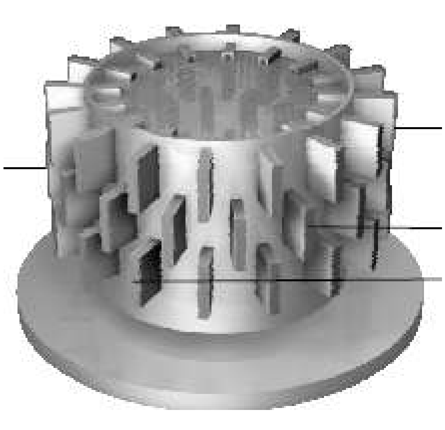

Figure 1: The USCT system.

For the reconstruction of tomographic images, it is crucial to know the positions of individual transducers accurate within the order of a wavelength. Even small positioning errors (in the range of tenths of millimeters) can lead to significant degradation of image quality. With respect to the number of transducers in the system, it is impractical to measure the distances between them manually.

There are many calibration techniques available in literature. Each technique was specifically developed for one of various technical fields, such as: microphone array processing, wireless sensor networks, a variety of different global navigation approaches (GPS, GLONASS, Galileo ...) [3], and others. But none of these methods was optimal for the USCT calibration task. A combination of features had to be satisfied: high accuracy ($<10^{-4}$m), no assumptions of sender-sender and receiver-receiver distances, dealing with extremely high number of measurements (>100,000) and unknown position parameters (>10,000). After a detailed analysis of all of these methods, the GPS method was chosen as a basis which is flexible enough to be altered in a way suitable for the USCT setup.

2. THE GPS PRINCIPLES

GPS allows the determination of positions of mobile receiver units anywhere on earth. This is achieved by processing coded signals received by the receiver unit from satellites orbiting the earth. These signals carry the information of the satellite's position on the orbit, and the time the signal was sent. The receiver solves a set of so-called pseudo-range equations. In such equation, the difference of the time in which the signal was sent and the time in which the signal was received (the pseudo-range) is expressed as a function of the satellite and receiver positions and their clock offsets from the universal GPS time [3]:

$$R_{s,r} = TOF_{s,r} + \tau_s + \tau_r + d_{TOF} \qquad (1)$$

$$TOF_{s,r} = r_{s,r}/c = \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2}\,/c \qquad (2)$$

where, $R_{s,r}$ is the pseudo-range of the satellite s and receiver r, $TOF_{s,r}$ is the time-of-flight of the signal from satellite to receiver, τ_s and τ_r are the internal clock offsets, d_{TOF} is the measurement noise, $r_{s,r}$ is the distance (range) from the satellite to the receiver, c is the speed of light, and x_s, y_s, z_s and x_r, y_r, z_r are the position coordinates of the satellite and the receiver unit respectively. The positions of the satellites and their clock offsets are known (or can be very accurately estimated) and so there are 4 unknowns to be solved for. This determines the fundamental number of at least 4 independent equations which need to be available, and therefore 4 satellites must be in view of the receiver. If more than 4 satellites are in view, more equations can be set up and the overdetermined set can be solved by means of the least squares error.

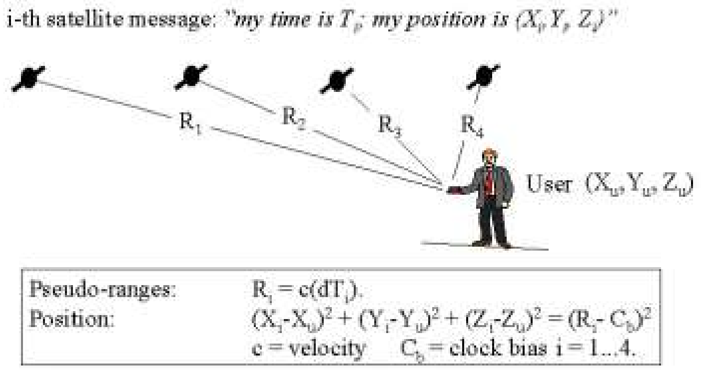

Figure 2: The principles of the GPS method.

The unknown coordinates and clock offset can be solved by the Gauss-Newton method (or some other minimization technique). Other GPS-specific factors, such as the multipath error, ionospheric error, and relativistic error are normally accounted for in the pseudo-range equations, but equations (1) and (2) are sufficient as a basis for the USCT system calibration.

3. THE USCT CALIBRATION METHOD

For the USCT calibration, a so-called empty measurement has to be made. In such a measurement, the tank is filled only with water. At any moment, only one transducer is sending an ultrasonic pulse, which travels through the water and reaches all receiving transducers.

Each receiver records this signal, called an A-scan (Figure 3). Then, another transmitter sends out an ultrasonic pulse and all receivers record the A-scans. This is repeated until all transmitters have been fired.

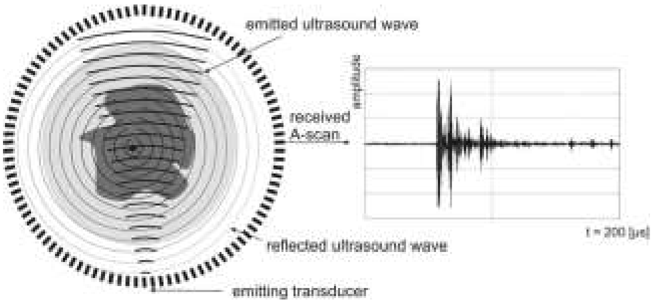

Figure 3: The USCT system (a view from the top).

In each A-scan, several pulses can be detected. The first corresponds to the direct path of the ultrasound wave from the sender to the receiver, whereas later pulses correspond to reflections from the tank walls. By detecting the position of the first pulse, we obtain a so called time-of-arrival value ($TOA_{s,r}$) for a particular sender-receiver combination.

In the derivation of the USCT calibration equations we can proceed analogically to the GPS solution. We can use the equations (1) and (2) if we alter a few variables. The value of speed of sound in water v has to be exchanged for speed of light c. The clock offsets τ_s and τ_r are an analogy of time delays introduced by the transducers and the electronics which process the signals. The $TOA_{s,r}$ value is equivalent to the pseudo-range $R_{s,r}$. Note that in this case neither the transmitter positions nor their delays are known. The only known parameter is the speed of sound v, which can be estimated from tabular values or computed if the temperature is known [2].

For solving the equation set, the Gauss-Newton method was chosen (as is the case in standard GPS devices). For this we need to introduce a set of transducer position estimate values (sender: x_{s0}, y_{s0}, z_{s0}, and receiver: x_{r0}, y_{r0}, z_{r0}) and estimate errors (sender: $\Delta x_s, \Delta y_s, \Delta z_s$, receiver: $\Delta x_r, \Delta y_r, \Delta z_r$). The actual positions are the sum of the estimates and errors: $(x_s, y_s, z_s) = (x_{s0} + \Delta x_s, y_{s0} + \Delta y_s, z_{s0} + \Delta z_s)$, $(x_r, y_r, z_r) = (x_{r0} + \Delta x_r, y_{r0} + \Delta y_r, z_{r0} + \Delta z_r)$. By substituting the above into (2) and expanding it into Taylor series (up to the 1st order) we obtain a linearized approximation of the time-of-flight value:

$$TOF_{s,r} = \sqrt{(x_{s0} + \Delta x_s - x_{r0} - \Delta x_r)^2 + (y_{s0} + \Delta y_s - y_{r0} - \Delta y_r)^2 + (z_{s0} + \Delta z_s - z_{r0} - \Delta z_r)^2} \Big/ c$$

$$\cong \frac{r_{s0,r0}}{v} + \frac{x_{s0} - x_{r0}}{v r_{s0,r0}} \Delta x_s - \frac{x_{s0} - x_{r0}}{v r_{s0,r0}} \Delta x_r + \frac{y_{s0} - y_{r0}}{v r_{s0,r0}} \Delta y_s - \frac{y_{s0} - y_{r0}}{v r_{s0,r0}} \Delta y_r + \frac{z_{s0} - z_{r0}}{v r_{s0,r0}} \Delta z_s - \frac{z_{s0} - z_{r0}}{v r_{s0,r0}} \Delta z_r$$

and $r_{s0,r0} = \sqrt{(x_{s0} - x_{r0})^2 + (y_{s0} - y_{r0})^2 + (z_{s0} - z_{r0})^2}$ is the sender-receiver distance estimate.

By plugging the above back into (1) we can formulate the overall difference between the estimated and detected time-of-arrival of each pulse:

$$\Delta TOA_{s,r} + d\tau_{s,r} = \frac{x_{s0} - x_{r0}}{v r_{s0,r0}} \Delta x_s + \frac{y_{s0} - y_{r0}}{v r_{s0,r0}} \Delta y_s + \frac{z_{s0} - z_{r0}}{v r_{s0,r0}} \Delta z_s + \Delta \tau_s$$
$$- \frac{x_{e0} - x_{r0}}{v r_{s0,r0}} \Delta x_r - \frac{y_{e0} - y_{r0}}{v r_{s0,r0}} \Delta y_r - \frac{z_{e0} - z_{r0}}{v r_{s0,r0}} \Delta z_r + \Delta \tau_r \quad (3)$$

We can now arrange a set of equations (3) (one for each sender-receiver combination) into a matrix representation: $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{x} = [\Delta x_{s1}, \Delta y_{s1}, \Delta z_{s1}, \Delta \tau_{s1}, \ldots \Delta x_{sN}, \Delta y_{sN}, \Delta z_{sN}, \Delta \tau_{sN}, \Delta x_{r1}, \Delta y_{r1}, \Delta z_{r1}, \Delta \tau_{r1}, \ldots \Delta x_{rM}, \Delta y_{rM}, \Delta z_{rM}, \Delta \tau_{rM}]^T$ is a column vector of the unknown estimate-error components, $\boldsymbol{b} = [\Delta TOA_{s1,r1} \ldots \Delta TOA_{s1s,rM}, \Delta TOA_{s2s,r1} \ldots \Delta TOA_{sNs,rM}]^T$ is the column vector of differences of estimated and detected TOA's, and $\boldsymbol{A}$ is the equation system matrix. The system can be solved via least squares method: $\boldsymbol{x} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$, where T denotes transpose and -1 matrix inversion.

In order to solve this equation set, we have to introduce some constraints otherwise it doesn't converge. If we analyze the problem closely, we find out that we are trying to solve the positions of several transducers (and their time delays) based only on the measurements of their relative distances. No information is given on the position and orientation of the USCT transducers relative to the coordinate system. If we imagine the correct solution of the equation system (the correct positions of all transducers) we could translate or rotate these positions (all at once) in any direction and under any angle and still obtain a correct solution of this system. Even though the system is heavily overdetermined (the ratio of number of equations to the number of unknowns is about 100) it is rank deficient and has therefore an infinite number of correct solutions.

We can constrain the system to one possible solution by introducing so-called anchors. In other calibration techniques, anchors are typically referred to as nodes of known positions. Although we are not sure where our transducers lie, we can simply set (anchor) the position of one transducer (s_1) to an unchangeable value (for example into the origin of the coordinate system: $s_1:\{0,0,0\}$). We can do this by adding an equation to the system expressing such a relation. Now, if we imagine the correct positions of the transducers again (having in mind, that we anchored one transducer) we cannot translate the system anymore, but we still can rotate it about this anchor. Therefore, other transducers must be anchored. But we cannot afford to do that, because fixing another transducer to a particular position will introduce an error, as we do not know this position in advance. Instead, we anchor only two out of the three coordinates (e.g.: x and z) which solves our problem $s_2:\{0,y,0\}$. This way the distance between s_1 and s_2 can still be adjusted by solving the least squares problem, but it also constraints the overall movement of other transducers. To stabilize the solution completely, we need to choose one more transducer, and anchor a different combination of two coordinates $s_3:\{x,0,0\}$. The anchoring can also be seen as choosing one coordinate system (out of an infinite number of possible systems) in which we solve the calibration problem.

4. SIMULATION RESULTS

In order to verify the method, a simulation study was done. At first, the size of the convergence region was tested. The initial position estimate values were derived from the ground truth position values by introducing an estimate error of various magnitudes. By solving the system of linearized equations (3) a set of error values (error residual) was obtained. Then by adding this error residual to the initial estimate, a new estimate was made. The process iterated 15 times. No measurement noise was assumed at first. The results can be seen in Figure 4. The convergence region is surprisingly large – in the magnitude of the diameter of the USCT system (20 cm). This means, that in presence of no noise, we can afford a large error in the first estimate and still arrive at the correct solution. The error residual minimum is limited by the precision of the used data type (double floating point).

Another part of the simulation was the introduction of measurement noise. By measurement noise we mean the inaccuracy of the pulse detection. It can be seen (Figure 5), that in order to satisfy the primary needs of the USCT calibration (accuracy of the calibrated positions of transducers should be within a tenth of a millimeter), it is essential to detect pulses with a higher accuracy than 10^{-7}s, which is just on the limits of the current pulse detection methods used in the USCT system. On the other hand, accuracy of the pulse detection algorithm is the only limiting factor of the overall calibration accuracy. Therefore the accuracy of the calibrated transducer positions can be expected to rise in the future with the employment of better pulse detection algorithms

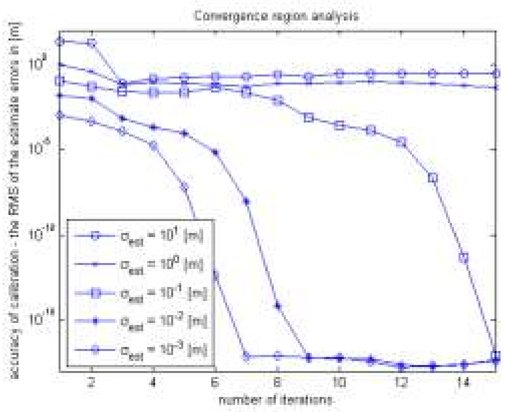

Figure 4: Convergence region analysis. The plot shows the calibration accuracy (RMS of the estimate errors) for different starting estimates. The standard deviation of the initial estimates is given in the legend (in meters). No measurement noise is assumed.

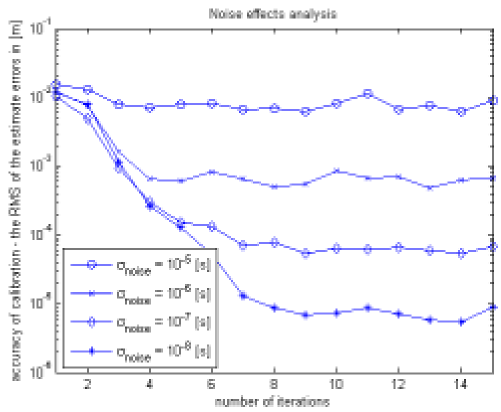

Figure 5: Noise effects analysis. This shows the calibration accuracy (RMS of the estimate errors) for different values of measurement noise. The standard deviation of the inaccuracies is given in the legend in seconds. The starting estimate accuracy is 10^{-2} m.

5. CONCLUSION

A new method was developed for calibration of a USCT system. For its flexibility, the GPS navigation principle was used as a core of this method. The main extension over GPS is that neither the transducers nor the receivers are assumed to be in known positions and all are calibrated at once. The calibration is self-contained – no additional "calibration phantoms" are needed. The accuracy of the method is only limited by the accuracy of the signal detection.

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