FULL WAVE SIMULATION USING FTDT METHOD

Martin Čížek
Doctoral Degree Programme (2), FEEC, BUT
E-mail: martin_cizek@phd.feec.vutbr.cz

Supervised by: Jiří Rozman
E-mail: rozman@feec.vutbr.cz

ABSTRACT

The paper deals with description of an example implementation of FTDT method for ultrasound field full wave simulation. Full wave simulations provide a vivid tool for studying both the spatial and temporal nature of an acoustic field. Presented solution is intended to be a part of software for ultrasound field modeling, currently in the state of development. As a model for the propagation of ultrasound, the Westervelt equation is used. Therefore, both linear and nonlinear propagation effects can be simulated.

1. INTRODUCTION

Numerical simulations are currently the best means of making predictions of nonlinear ultrasound propagation. One of the numerical techniques suitable for providing full-wave solutions to the propagation problem is the finite-difference time-domain (FTDT) method. In this work, an implementation of FTDT into an application for ultrasound field modeling is described.

2. MODEL WAVE EQUATION

Soft tissue media are commonly modeled as thermoviscuous fluids. The Westervelt equation was chosen as a model equation for the propagation of ultrasound in a thermoviscous fluid. It is derived from the equation of fluid motion by keeping up to quadratic order terms [1]. The equation can be written in the following form:

$$\nabla p - \frac{1}{c_0^2}\frac{\partial p}{\partial t} + \frac{\delta}{c_0^4}\frac{\partial p}{\partial t} + \frac{\beta}{\rho c_0^4}\frac{\partial p^2}{\partial t} =) \qquad (1)$$

where p is the acoustic pressure, ρ_0 and c_0 are the ambient density and ultrasound speed, δ is the diffusivity of ultrasound, $\beta = + \frac{B}{A}$ is the coefficient of nonlinearity. B/A is the parameter of nonlinearity of the fluid.

The first two terms in (1) describe linear lossless wave propagation. The third term describes the loss due to the viscosity and thermal conduction of the fluid. Finally, the nonlinear distortion of the traveling wave due to finite-amplitude effects is described by the fourth term.

3. USING THE FTDT METHOD

FTDT method approximates the spatial and temporal partial derivatives with discrete differences, which can be obtained from Taylor series expansions about each node of the computational grid.

Let's consider a grid with two spatial dimensions and uniform spacing of Δx and Δr, indexed by (i, j). Temporal dimension has a uniform spacing of Δt and is indexed by n.

Temporal derivatives present in the absorption and nonlinear terms can be calculated to second order accuracy, using backward-time differences as follows:

$$
\begin{aligned}
\frac{\partial^2 p}{\partial t^2} &\approx \frac{1}{\Delta t^2}\left(p_{i,j}^{n+1} - 2p_{i,j}^{n} + p_{i,j}^{n-1}\right)\\
\frac{\partial^3 p}{\partial t^3} &\approx \frac{1}{(2\Delta t)^3}\left(6p_{i,j}^{n} - 23p_{i,j}^{n-1} + 34p_{i,j}^{n-2} - 24p_{i,j}^{n-3} + 8p_{i,j}^{n-4} - p_{i,j}^{n-5}\right)
\end{aligned}
\qquad (2)
$$

The spatial differences can be obtained using a fourth order accurate, centered differencing. The example for r-dimension is shown:

$$
\begin{aligned}
\frac{\partial p}{\partial r} &\approx \frac{1}{12\Delta r}\left(-p_{i,j+2}^{n} + 8p_{i,j+1}^{n} - 8p_{i,j-1}^{n} + p_{i,j-2}^{n}\right)\\
\frac{\partial^2 p}{\partial r^2} &\approx \frac{1}{12(\Delta r)^2}\left(-p_{i,j+2}^{n} + 16p_{i,j+1}^{n} - 30p_{i,j}^{n} + 16p_{i,j-1}^{n} - p_{i,j-2}^{n}\right)
\end{aligned}
\qquad (3)
$$

According to [3], this approach reduces the effects of numerical dispersion. Discussion on numerical stability and dispersion can be found in [2].

3.1. FTDT ALGORITHM IMPLEMENTATION IN MATLAB

The FTDT algorithm was implemented and evaluated in MatLab environment. To achieve optimal application code, attention has to be paid to minimizing memory usage. The number of performed memory transfers should also be as low as possible.

FTDT algorithm is often likely to be treated as a solution to a system of N equations. Let's consider a spatial grid with dimensions of N_x by N_r, K is the number of recent time steps allocated in the memory for temporal derivatives calculations.

In case of using traditional approach for solving a system of N equations in MatLab, $(N_x.N_r.K)$ by $(N_x.N_r.K)$ square matrix needs to be allocated in the memory for determinant computation, etc. This would result in a great memory consumption and slower speed of mathematic operations carried out with such a large matrix.

A less memory consuming approach chosen in the presented application is based on direct calculation of spatial nodes based on transforming equations (2) and (3) into matrix mathematics. In this case, only N_x by N_r matrix needs to be allocated for each of the recent K time steps. Additionally, advantages of optimized matrix operations supported by MatLab environment can be taken.

Looking at the equations (2) and (3), it is obvious that six previously calculated grids need to be kept in the memory for correct computation of the current spatial grid. The newly calculated grid consequently replaces the oldest one in the circular buffer as seen in fig. 1, while the other grids shift one buffer position further.

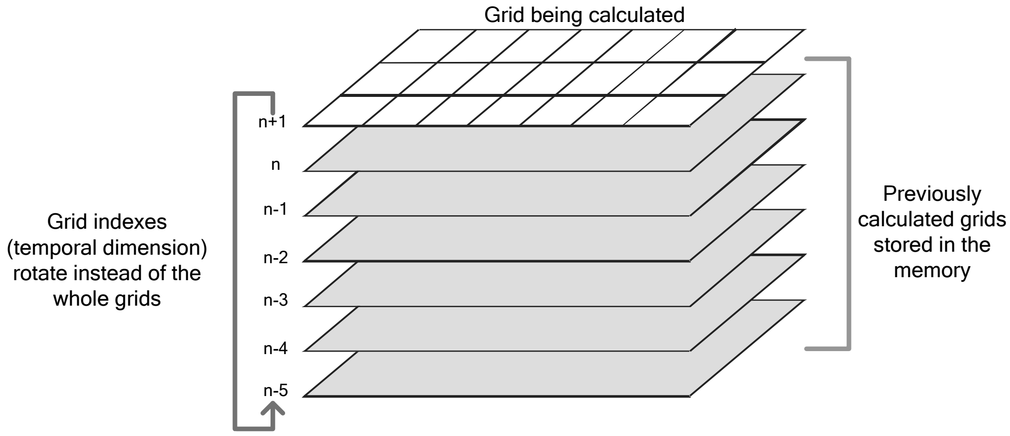

Fig. 1: Circular buffer used for storing recently computed data

To improve the speed of computation, no data physically rotate in the circular buffer. The effect of rotation is achieved only by reorganizing the indexes of the circular buffer elements.

4. RESULTS

A 1 MHz sinusoidal burst of 6 cycles modulated by a Gaussian envelope in time was used as the source signal in all the following simulation examples.

Simulation in fig. 2 shows a reflection of the pulse at a boundary of two media with different ultrasound propagation speeds c_{01} = 1600 ms^{-1} and c_{02} = 1300 ms^{-1}. Initial pressure amplitude P_0 = 100 kPa. Temporal dimension spacing $\Delta t = 5$ ns, spatial spacing Δr = 25 µm. Only linear propagation of ultrasound was modeled. The model in fig. 3 is an extension to the previous using a simulation in 2D environment.

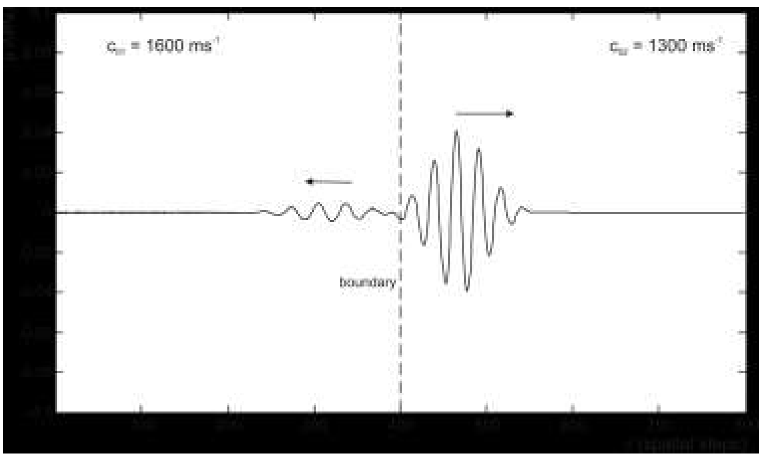

Fig. 2: Reflection of a pulse on a boundary of two media with different propagation speeds.

Finally, propagation in a highly nonlinear media is simulated in fig. 4. Gradual accumulation of the energy of the pulse into 2nd and 3rd order harmonics can be clearly seen.

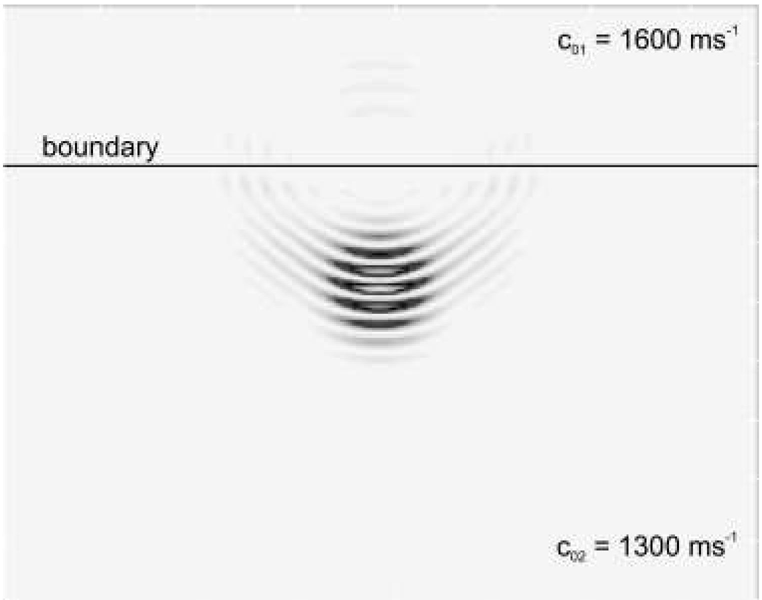

Fig. 3: Ultrasound propagation 2D model

In the simulation in fig. 4, nonlinear propagation effects are highly exaggerated, the purpose is to demonstrate model's ability to handle nonlinearities. Coefficient of nonlinearity β is equal to 1000 in this case. In the real world, β value is several hundred times lower (for example approx. 5 in soft tissues).

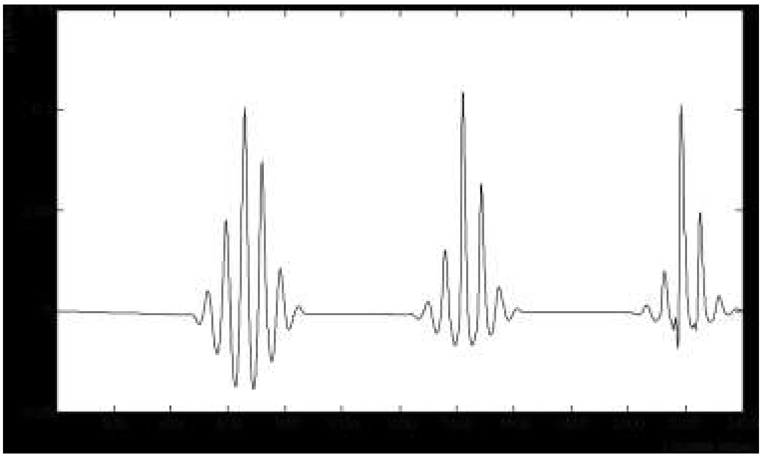

Fig. 4: Nonlinear distortion of pulses at different travel distances from the source (exaggerated for clearness).

5. CONCLUSION

Presented evaluations show that FDTD method is easily applicable for solving both linear and nonlinear ultrasound propagation problems. Designed algorithm is a part of a project which topic is study of nonlinear effects in diagnostic ultrasound applications. Future work will include extending the model into 3D. Calculation of heating effects due to higher order harmonics absorption in biological tissues is also planned.

Another future goal is to encapsulate the code in an user friendly software application, which will serve as an aid in study of nonlinear effects in diagnostic ultrasound propagation.

ACKNOWLEDGEMENTS

This paper has been prepared with support of the Czech Ministry of Education grant No. 488/2007 and research plan MSM 0021630513.

REFERENCES

[1] HAMILTON, M. F., MOFREY, C. L. *Model equations*. Nonlinear Acoustics, Chap. 3, Academic Press, San Diego, 1998

[2] NAKAMURA, S. *Applied Numerical Methods with Software.* Pretince Hall, New Jersey, 1991

[3] HALLAJ, I. M., CLEVELAND, R. O. *FTDT simulation of fininte-amplitude pressure and temperature fields for biomedical ultrasound.* Acoustics Research Letters Online, Accoustical Society of America, March 9, 1999, ISSN 1529-7853

[4] HUMPHREY, V. F. *Non-linear propagation for medical imaging*. In WCU 2003, Paris, September 7-10, 2003