DERIVATION OF GENERAL RELATION FOR LINEAR NETWORK TRANSFORMATIONS AND DISCUSSION OF ITS MATHEMATICAL CORRECTNESS

Jan Hrdý
Master Degree Programme (5), FEEC BUT
E-mail: xhrdyj00@stud.feec.vutbr.cz

Supervised by: Jaromír Brzobohatý
E-mail: brzoboh@feec.vutbr.cz

ABSTRACT

In the contribution the general relation between the original and the transformed network matrices for various types of linear network transformations is derived and its mathematical correctness is discussed.

1. INTRODUCTION

Most of the cases of linear network transformations can be represented by a system consisting of linear *n*-port and active linear transformation (*m*+*n*)-port (Fig. 1). Symbols **W'**, **W** and **T** denotes an arbitrary network matrices of the *n*-port, *m*-port and (*m*+*n*)-port. The matrices **W'**, **W** and **T** are square matrices of the orders *n*, *m* and (*m*+*n*) according to the theory of *n*-ports.

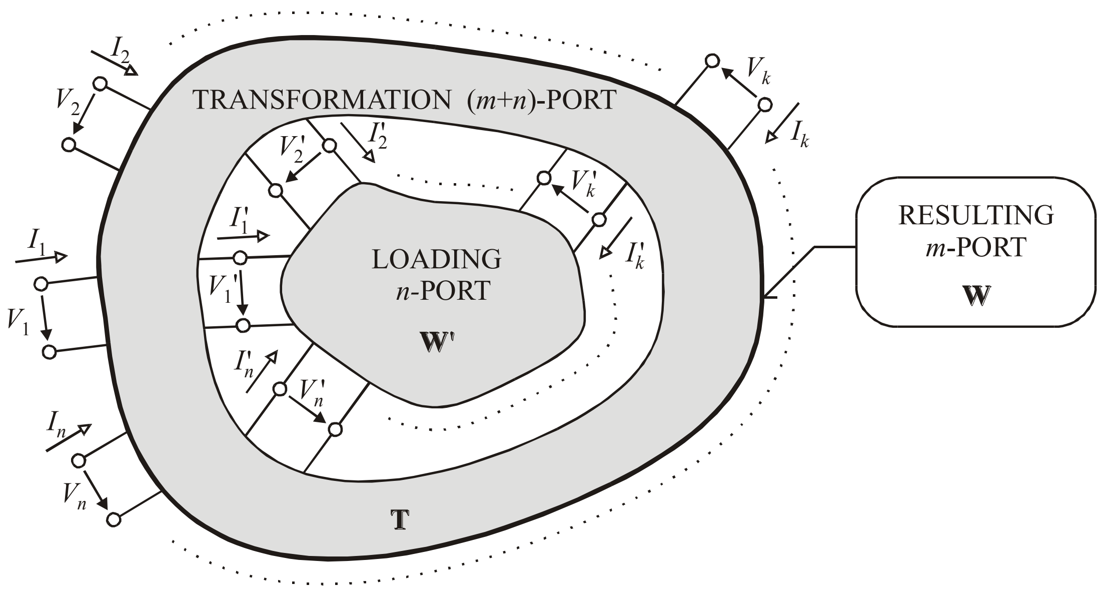

Figure 1: Symbolic diagram of the linear transformation system.

The main problem of the analysis of the whole system behaviour and also of the synthesis procedure is to find the general transformational relation

$$\mathbf{W} = \quad \mathbf{W}\ \mathbf{T} \tag{1}$$

for the given type of matrices **W'**, **W** and **T** and to prove its mathematical correctness.

2. DERIVATION OF GENERAL TRANSFORMATIONAL RELATIONS AMONG THE NETWORK MATRICES W', W AND T

2.1. ORIGINAL DERIVATION OF TRANSFORMATION RELATIONS

Let us first write the equations of the system in Figure 1 in the matrix form. For the loading n-port we have

$$\mathbf{X}_1' = \mathbf{W}\mathbf{X}_2, \tag{2}$$

where

$$\mathbf{X}_1'\ \mathbf{X}_2' \quad = \mathbf{P}\ \ \mathbf{V}\ \ \mathbf{I} \tag{3}$$

and

$$\mathbf{V}' = \quad {}' \ , \mathbf{I}' = \quad {}' \ . \tag{4}$$

Matrix **P'** is the permutation matrix (a square matrix having in each row and column only one nonzero element which is equal 1 or -1) which providing the possibility of obtaining any required permutation of the port variables from the defined basic permutation. For the whole system as an m-port we have an equations that are equivalent to the equations (2), (3) and (4) for variables non-signed by a comma. And for the transformation (m+n)-port we have

$$\mathbf{X}_{T1} = \mathbf{T}\mathbf{X}_{2}, \tag{5}$$

where

$$\mathbf{X}_{T1}, \mathbf{X}_{T2}\ ^{T} = \mathbf{P}\ \ \mathbf{V}\ \mathbf{I}\ \mathbf{V}'\ \mathbf{I}' \ . \tag{6}$$

Using relative complicated algebraic approach [1] we obtain the relation (1) in the form of the so-called double linear transformation

$$\mathbf{W} = \mathbf{F}_{11}(\mathbf{1} - \mathbf{W}'\mathbf{F}_{21})^{-1}\mathbf{W}'\mathbf{F}_{22} + \mathbf{F}_{12} \tag{7}$$

where

$$\mathbf{F} = -\tilde{\mathbf{P}} + \mathbf{T}\tilde{\mathbf{P}} \qquad \tilde{\mathbf{P}} - \mathbf{T}\tilde{\mathbf{P}} \tag{8}$$

and

$$\tilde{\mathbf{P}} = \mathbf{P} \qquad \lfloor \mathbf{P} \quad \mathbf{P}^{\ 1} \rceil \mathbf{P}_{o}\,. \tag{9}$$

The matrix **F** is an auxiliary square matrix of an order (m+n). The matrices $\mathbf{F}_{11}$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\mathbf{F}_{22}$ are sub matrices of matrix **F** and have dimensions $m \times n$, $m \times m$, $n \times n$ and $n \times m$. Matrix $\tilde{\mathbf{P}}$ is an auxiliary square permutation matrix of an order 2(m+n). Matrices $\tilde{\mathbf{P}}_{11}$, $\tilde{\mathbf{P}}_{12}$, $\tilde{\mathbf{P}}_{21}$ and $\tilde{\mathbf{P}}_{22}$ are square sub matrices of matrix $\tilde{\mathbf{P}}$ of an order (m+n).

2.2. NOVEL DERIVATION OF TRANSFORMATIONAL RELATIONS TO SHOW MATHEMATICAL CORRECTNESS OF EQUATION (7)

To solve this task, it is convenient to start from the relations among the port variables expressed in the form (so-called basic permutation)

$$\mathbf{V} = \mathbf{WI}\,,\ \mathbf{V}' = \mathbf{W}\,\mathbf{I}\ \text{ and } \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} \ \overbrace{\begin{bmatrix} \mathbf{T} & \mathbf{T} \\ \mathbf{T} & \mathbf{T} \end{bmatrix}}^{\mathrm{T}} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix}. \qquad (10a\text{-}c)$$

The equations (10a-c) are in fact only special cases of general equation of mathematical linear transformation (see [2], chapter 8). Now we derivate the transformation relation among the matrices **W'**, **W** and **T** exactly according to the mathematical theory of linear transformation.

In the first step we rewrite Eq. (10c) to two independent matrix equations (mathematical correctness of this operation is evident)

$$\mathbf{V} = \ \mathbf{T}_{11} \quad \mathbf{T}_{12} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix},\ \mathbf{I}' = \ \mathbf{T}_{21} \quad \mathbf{T}_{22} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix}. \qquad (11a,b)$$

In the next step we set together the linear transformations (10b) and (11b) and after rearrangement we obtain the composition of linear transformation equation in the form

$$\mathbf{V}' = \ \mathbf{1}\ \mathbf{W}\,\mathbf{T}_{21} \quad \mathbf{W}'\mathbf{T}_{22}\mathbf{I}\,, \qquad (12)$$

and in parallel we set together equations (11a) and (12) and we obtain

$$\mathbf{V} = \ \mathbf{T}_{11}\ \ \mathbf{1}\ \mathbf{W}\,\mathbf{T}_{21} \quad \mathbf{W}\,\mathbf{T}_{22} + \mathbf{T}_{12}\ \ \mathbf{I}\,. \qquad (13)$$

The linear transformations (12) and (13) comply with the requirements of the theorem 8.1.2 [2], thus relations (12) and (13) are mathematical correctness.

In the last step we get the final relation from Eq. (13) in the form (proof of the mathematical correctness is evident)

$$\mathbf{W} = \mathbf{T}_{11}\ \ \mathbf{1}\ \mathbf{W}\,\mathbf{T}_{21} \quad \mathbf{W}\,\mathbf{T}_{22} + \mathbf{T}_{12}\,. \qquad (14)$$

3. CONCLUSION

It is obvious that the equation (14) is from a viewpoint of the theory of linear network transformation a special case of the equation (7) and the matrices $\mathbf{T}_{11}$, $\mathbf{T}_{12}$, $\mathbf{T}_{21}$ and $\mathbf{T}_{22}$ are the special cases of the matrices $\mathbf{F}_{11}$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\mathbf{F}_{22}$. On the other hand the equations (14) and (7) are from a viewpoint of the mathematical theory of linear transformation equivalent. The matrices **T** and **F**, likewise, are equivalent (their elements have different numerical value, but it does not matter). A mathematical correctness of the linear transformation represented by Eq. (14) was proved in the chapter 2.2. From this point of view the double linear transformation represented by the equations (7) and (8) can be considered as a mathematical correctness.

REFERENCES

[1] Brzobohatý, J., Pospíšil, J.: A Generalized Approach to Linear Network Transformations and Their Modelling. *In*: Proc. ECCTD'83, Stuttgart, 1983, pp. 65-67.

[2] Howard, A.: Elementary Linear Algebra., John Wiley & Sons, Inc., New York 1994.