MULTI-CHANNEL QUEUING PROBLEMS APPROACH

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ABSTRACT

The paper deals with one problem of multi channel queuing problems. It shows that multi-channel queuing model without losses can be used to solve queuing problems. The service channels are composed of parallel stations; each one of the stations can deliver the same type of service and is equipped with the same type of facilities.

1 INTRODUCTION

There are two types of multi channel problems. The first type occurs when the system has more service centers, and the queues to every of them are isolated and an element cannot pass from one queue to the other. The second type of waiting-queue problem is said to be of multiple exponential channels or of several service channels in parallel, when the element in queue can be served equally well by more than one station.

2 MULTIPLE-CHANNEL QUEUING PROBLEMS

The queuing problems in multi-channel problems should be considered rather as several problems of the single-channel type. Fig. 1 illustrates a valid case; the formation of each queue is independent from the others. When an element has selected the concrete queue, it becomes part of the single-channel system.

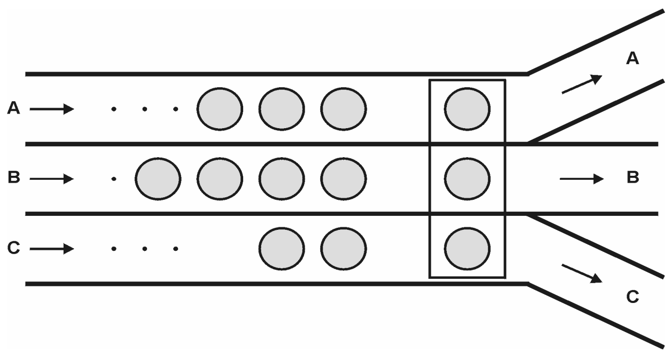

Fig. 1: *Multiple-channel queuing problem*

The probability of the arrivals in queue A is independent from the probability of the arrivals in queue B (and C) due to different characteristics of the routes. On the other hand, in multiple exponential channels type each one of the stations can deliver the same type of service and is equipped with the same type of facilities. The element which selects one station makes this decision without any external pressure from anywhere. Due to this fact, the queue is single. The single queue (line) usually breaks into smaller queues in front of each station. Fig. 2 schematically shows the case of a single line (which has its mean rate λ) that randomly scatters itself toward four stations (S = 4), each of which has an equal mean service rate μ.

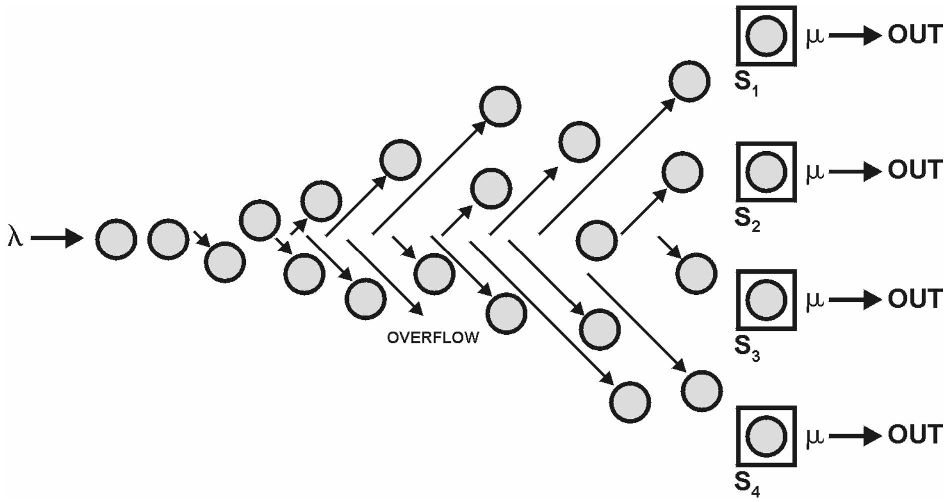

Fig. 2: *The case of multi-parallel servicing stations*

2.1 MULTIPLE-CHANNEL QUEUING PROBLEMS WITHOUT LOSSES (M/M/S)

When the service channels are composed of S parallel stations, the state of the system n (number of elements present in the system at a certain moment) can assume one of these values:

1) $n \leq S$ there is no queue (line) because all elements are being served.

2) $n > S$ a queue is formed of the length $n - S$ (provided the formation of the waiting line is permissible by the nature of the problem).

We define the relationship between the mean servicing rate and the stations as follows:

$\mu_n = \mu S$ for $n > S$ where μ is a mean service rate of every channel.

The utilization factor ρ_S for the whole system is the ratio between the mean arrival rate λ and the maximum possible rate of service of all the channels; hence

$$\rho_S = \frac{\lambda}{\mu S} \tag{1}$$

As pointed out earlier, in multi-channel problems (S stations) each servicing point has a mean rate μ and the elements come at a mean arrival rate λ. The probability that there are n elements in system, when it is equipped with two or more stations, and n is less than S, is:

$$P_{S_n} = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_{S_0} \quad \text{for } n = 0, 1, \ . \ . \ . \ , S\text{-}1 \quad \text{i.e.} \quad n < S \tag{2}$$

When the number of elements equals or is greater than the number of stations *S*, this probability becomes:

$$P_{S_n} = \frac{1}{S!\,S^{n-S}}\left(\frac{\lambda}{\mu}\right)^n P_{S_0} \quad \text{for} \quad n \ge S \tag{3}$$

The probability of having no elements in a multi-channel system is:

$$P_{S_0} = \left[\sum_{i=0}^{S-1} \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i + \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^S \frac{\mu S}{\mu S - \lambda}\right]^{-1} \tag{4}$$

The above formulas can be applied only if $\mu S > \lambda$ or $\rho_S < 1$. In the opposite case $(\mu S \le \lambda,, \rho_S \ge 1)$ the queue will increase indefinitely in size.

In the case of single-channel, the probability the arrival will not have to wait for service is $P_0 = 1 - \frac{\lambda}{\mu}$ while the probability that an arrival has to wait for service is:

$$P_n = 1 - P_0 = 1 - \left(1 - \frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \tag{5}$$

In the multi-channel system, the probability that an element approaching the stations has to wait to be serviced coincides with the probability that there is *S* or more, elements in the system:

$$P_{S_{n=}} = \frac{\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!\,(\mu S - \lambda)} P_{S_0} \tag{6}$$

where n' is any (number of elements) from S (included) up to n .

The mean length of the waiting line (number of units –*m*'**)**, or average queue length, excluding the elements under service, is obtained by multiplying equation (4) by ratio $\frac{\lambda}{\mu S - \lambda}$, which yields:

$$m_Q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(\mu S - \lambda)^2} P_{S_0} \tag{7}$$

The average number of elements in the multi-channel system is:

$$m_S = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(\mu S-\lambda)^2}P_{S_0} + \frac{\lambda}{\mu} \tag{8}$$

The average waiting time of an element which has arrived in the system is:

$$W_Q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(\mu S-\lambda)^2}P_{S_0} \tag{9}$$

The total average time that an element (arrival) spends in the system is:

$$W_S = \frac{\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(\mu S-\lambda)^2}P_{S_0} + \frac{1}{\mu} \tag{10}$$

3 CONCLUSIONS

In multi channel queuing problems, each one of the stations can deliver the same type of service and is equipped with the same type of facilities. the queue usually breaks into smaller single queues in front of each station. The element which selects one station makes this decision without any external pressure from anywhere. Due to this fact, the queue is single.

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