MODELLING AND LINEAR CONTROL OF THE CRANE

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ABSTRACT

This article deals with a modelling of an overhead crane system with a DC drive. There is assumed the linear crane system in this article. System is described by mechanical equations and the model of the system is designed. In the second part of the article the proposal of state-space controller is described. The modeling of the system is executed in Matlab-Simulink.

1 INTRODUCTION

Overhead cranes are used in numerous industrial applications, such as the loading and unloading of containers, nuclear waste handling facilities, factory automation and basically in any industry which requires heavy goods to be lifted and moved. The crane systems are desired to be able to move to the required positions as fast and as accurate as possible while placing the payload at the appropriate position [1].

The main goal in solving the crane system is to design the controller to reduce the load swinging and to get high position accuracy.

The overhead crane system is typical nonlinear system, but the crane system described in this article is assumed to be linear. To facilitate modelling of the system is assumed that the system possesses the following characteristics:

- The payload mass and the crane trolley are assumed to be connected by a mass-less, rigid link during horizontal movements.
- The angular position and velocity of the payload mass and the rectilinear position and velocity of the crane trolley are assumed to be measurable.
- The payload mass is assumed to be concentrated at a point and the value of this mass is known exactly. Additionally, the value of the trolley mass is known exactly.
- The hinged joint, where the payload link connects to the crane trolley, is assumed to be frictionless.[2]

2 MODELLING OF THE SYSTEM

2.1 MODEL OF THE CRANE

The model of the crane is shown in the figure 1. In the picture M is the mass of the load, m is the mass of the cart, l is the length of the rope, θ is the angle and F is the force acting on the crane.

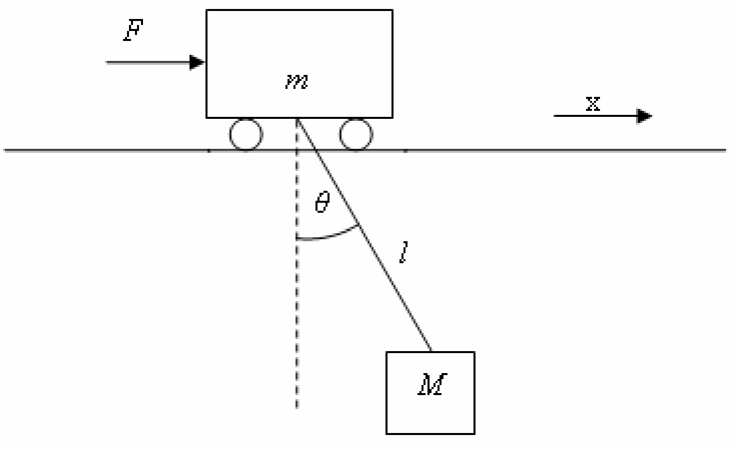


Fig. 1: *The crane system*

The energy of the system in figure 1 is described by following equations.

Kinetic energy of the crane:

$$E = \frac{1}{2} M \left[\dot{x} + l \cdot \dot{\theta} \right]^2 + \frac{m}{2} \dot{x}^2 \tag{1}$$

Potential energy of the crane:

$$V = -Mgl \cdot \cos(\theta) \tag{2}$$

The Lagrangian equation of the system is found by combining (1) and (2):

$$L = E - V \tag{3}$$

Then the equations of motion are:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}} L \right) - \frac{\partial}{\partial x} L &= F \\ \frac{d}{dt}\left(\frac{\partial}{\partial \dot{\theta}} L \right) - \frac{\partial}{\partial \theta} L &= 0 \end{aligned} \tag{4, 5}$$

where F is the force that is applied by the motor to the crane.

It is supposed that the value of the angle θ is close to zero. Then:

$$\sin(\theta) \approx 0, \cos(\theta) \approx 1 \tag{6, 7}$$

Because of these conditions the equations can be rewrite in form:

$$(M+m)\ddot{x} + Ml\ddot{\theta} = F$$
$$\ddot{x} + l\ddot{\theta} + g\theta = 0 \tag{8, 9}$$

These equations can be rewrite in state equations:

$$\mathbf{x}' = \mathbf{Ax} + \mathbf{Bu} \tag{10}$$

$$\mathbf{y} = \mathbf{Cx} \tag{11}$$

Equation (10) with all variables is:

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \dfrac{gM}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{g}{l}\left(1+\dfrac{M}{m}\right) & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m} \\ 0 \\ -\dfrac{1}{lm} \end{bmatrix} F \tag{12}$$

where x is the cart position, x' is the cart velocity, θ is the swing angle and θ' is the swing velocity.

2.2 MODEL OF THE DC MOTOR

The DC motor produces the force F which effects on the crane. The DC machine is described by equations:

$$\frac{d}{dt} i_a = \frac{1}{L_a}\left(U_a - R_a i_a - C_e \omega\right) \tag{13}$$

$$\frac{d}{dt}\omega = \frac{Ce}{J} i_a \tag{14}$$

where i_a is the armature current, U_a is the applied armature voltage, L_a is the armature inductance, R_a is the armature resistance, C_e is the motor constant, J is the moment of inertia of motor load and ω is the angular velocity of rotor shaft.

To obtain the force F acting on the crane it is necessary to determine the torque developed on the motor shaft. The torque has to be converted to the linear force F.

3 CONTROLLER DESIGN

To design the state-space controller it is used the method of the pole placement of the pole-placement of the characteristic equation of the closed loop. The new pole-placement changes the dynamic behaviour of the closed loop. State-space controller makes linear feedback from all state variables [3].

$$\det |p\mathbf{I} - \mathbf{A} + \mathbf{BR}| = \prod_{i=1}^{n}\left(p - \lambda_i\right) \tag{15}$$

4 SIMULATION AND RESULTS

The system described in chapter two and the state-space controller described in chapter three is used for simulation in Matlab-Simulink. There are used the parameters of the real system in this simulation. For the controller design it is assumed that all state variables are measurable. The figure 2 shows the model of the system with state-space controller used for simulation. The block called System model contains the model of the crane and the model of the DC motor. The figure 3 represents the step response of the system variables (cart position, swing angle) without state-space controller and the figure 4 shows the step response of the system with state-space controller.

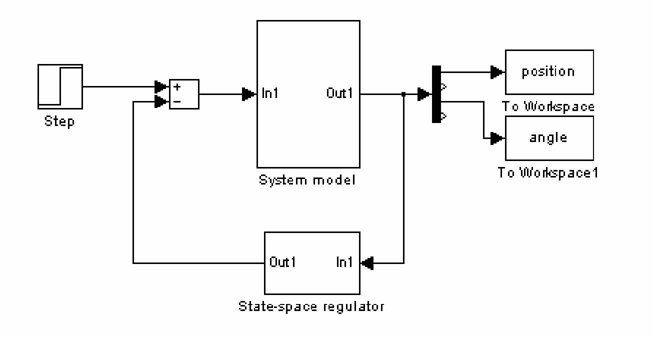
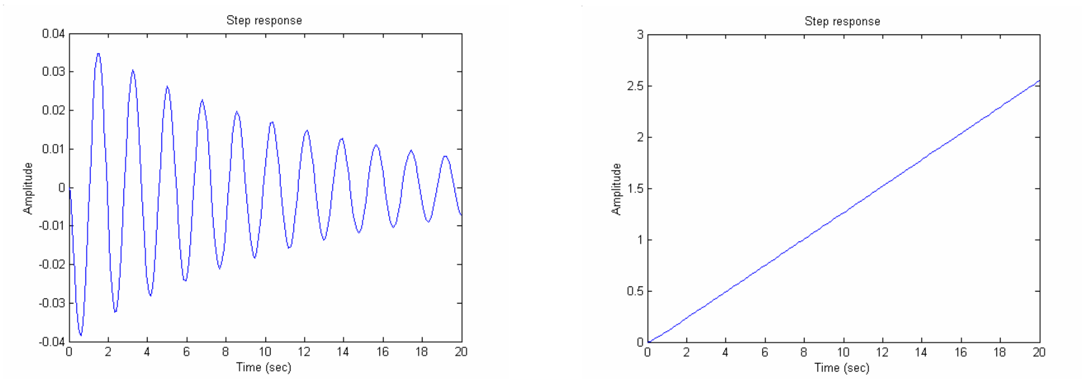
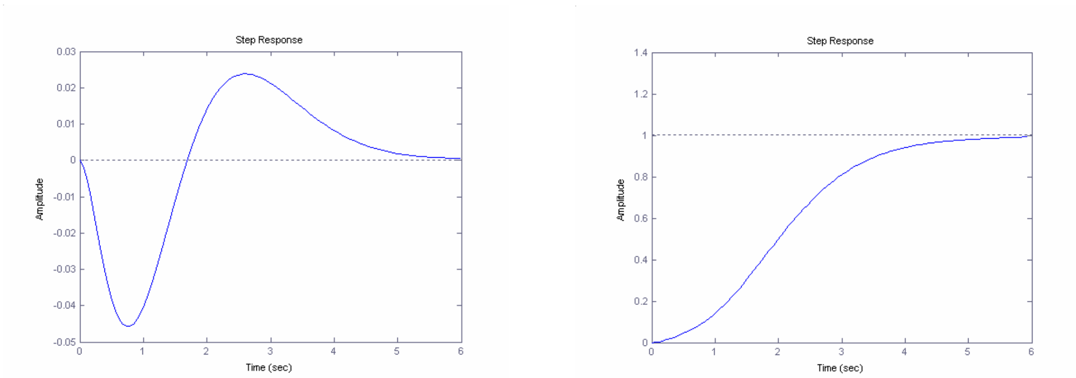


Fig. 2: *Model of the system in Matlab-Simulink*



a) *b)*

Fig. 3: *Step response of system without state-space controller: a)swing angle, b)position of the cart*



a) *b)*

Fig. 4: *Step response of system with state-space controller: a)swing angle, b)position of the cart*

5 CONCLUSION

The behaviour of the crane system with DC motor is investigated in this article. The system described in chapter 2 is assumed to be linear. The design of the state-space controller is described in chapter 3. The state-space controller is designed to be optimal for this application.

It is showed in figure 4a and 4b that the state-space controller designed for controlling the crane system is suitable to control complex system as this system is. The figures show the step response of swing angle and step response of cart position. In comparison with figure 3a and 3b we can see that the system with state-space controller acquires the desired position slightly in short time and the swing angle is damped in short time without any marked oscillations.

The behaviour of this modelled system is similar to the behaviour of the real system with the same parameters.

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