

# TESTING OF BASIC PARAMETERS SETUP AND VARIATIONS OF THE PARTICLE SWARM GLOBAL OPTIMIZATION TECHNIQUE

Ing. Lukáš OLIVA, Doctoral Degree Programme (1)  
Dept. of Radio Electronics, FEEC, BUT  
E-mail: xoliva02@stud.feec.vutbr.cz

Supervised by: Prof. Zbyněk Raida

## ABSTRACT

This article introduces an original way of increasing the probability of the convergence of classical Particle Swarm Optimization, a modern global optimization algorithm. The algorithm was tested and the test results on three mathematical functions known to be difficult to find their minimum are presented for three different parameter ranges. The results of tests done for three typical parameters setups are compared with those gained with use of the usual Particle Swarm Optimization form.

## 1 INTRODUCTION

The Particle Swarm Optimization (PSO) is a member of global optimization techniques. PSO was developed in the 90's as a result of the bee swarm organization observations and its behavior when searching the area for flowers. The optimization technique derived from these observations considers the minimization of the fitness function (criterion function) to be equivalent to the searching of the greatest density of flowers. The optimized parameters form the searched  $N$  - dimensional space. One parameter setup form possible solution is called particle or agent and represents one bee positioned in the point characterized by its parameter setup. In the first step the particles characterized by the position on each axis are randomly placed so that the parameter values are randomly generated. So are randomly generated the magnitude of the speed elements in all axes directions. In the second step the personal ( $P_{best}$ ) and global ( $G_{best}$ ) minimum fitness function values are evaluated. Then the speed components are updated according to the position of  $G_{best}$  (the lowest value of the swarm) and  $P_{best}$  (generally different for each particle) the influence of  $P_{best}$  and  $G_{best}$  is expressed by the weight coefficients. In the third step the position of each particle is updated according to the value of the speed components. In this step, it is possible some particles can overflow the permitted parameter boundaries - reach the restricted parameter area. There several possibilities can occur according to the setup of the optimization method. In the literature, three known wall-types are described:

- a) reflecting walls causing "reflection" in the sense of the law of reflection,

b) absorbing wall that behave as the ideal kinetic energy absorber and stop the particle and

c) invisible causing suppression of the particle if being out of the parameter boundary range.

The second and third steps are being repeated until the stop condition occurs. This condition can be formed as reaching some sufficiently small value of the fitness function or reaching some selected maximum number of iterations. Then the most suitable value is the  $G_{best}$ .

## 2 PSO SETUP

The setup of the each PSO optimization process consists of the fitness function evaluation (e.g. sum of squares), wall-type setup, condition of the loop interruption and the constant setup. Different parameter setups were evaluated for different optimization problems in the articles concerning the PSO method. The typical values of the weight coefficients described in [1] are used to find minimum of three different functions known to be uneasy to find their minimum [1] Rosenbrock (3), Rastigrin (2) and Griewank (1) functions of the second order. All of them are known to have their minimum in point  $[0, 0]$  except for the first one which has its minimum in  $[1; 1]$ .

$$f(x) = 1/4000 \cdot (x_1^2 + x_2^2) - \cos(x_1) \cos(x_2 / \sqrt{2}) + 1 \quad (1)$$

$$f(x) = (x_1^2 + x_2^2) - 10 \cos(2\pi x_1) - 10 \cos(2\pi x_2) + 20 \quad (2)$$

$$f(x) = 100(x_2 - x_1^2)^2 + (x_1 - 1)^2 \quad (3)$$

These functions were optimized by the traditional methods with various wall and weight coefficients setups. If certain condition selections (walls and weight coefficients) the PSO tended to converge frequently slowly or insufficiently. This can be explained in some cases as a false attractor. This means that the important part of swarm is attracted to the one minimum but not the global one. To prevent this phenomenon the basic PSO position update formula was changed.

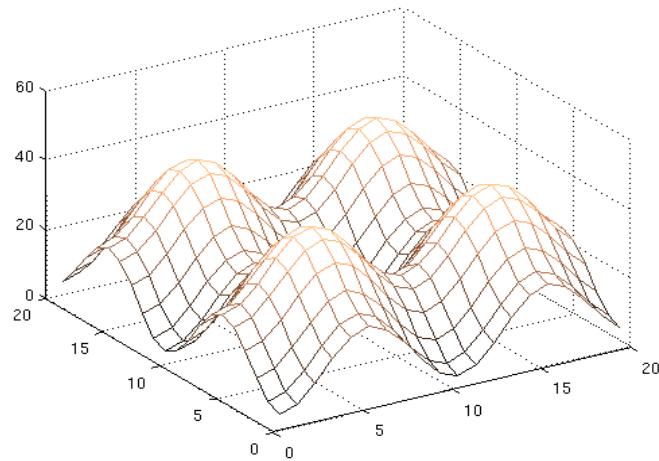


Fig 1: Rastigrin function for  $\langle -1; 1 \rangle x \langle -1; 1 \rangle$

### 3 UPDATE FORMULA MODIFICATION

The PSO original velocity update formula can be expressed in two ways. One form (4) use two constants for influencing the optimization setup ( $K$ ,  $\varphi_1$ ,  $\varphi_2$ ), the other reduces three constants to a couple ( $c_1$ ,  $c_2$ ) (5).

$$v_n = K(v_n + \varphi_1 \cdot rand() * (p_{best,n} - x_n) + \varphi_2 \cdot rand() * (g_{best,n} - x_n)) \quad (4)$$

$$v_n = (v_n + c_1 \cdot rand() * (p_{best,n} - x_n) + c_2 \cdot rand() * (g_{best,n} - x_n)) \quad (5)$$

$$P_n = v_n \delta_t \quad (6)$$

To improve the fitness function convergence the position update formula (6), where  $\delta_t$  is constant time step was modified to (7) and two new constants  $k_1$ ,  $k_2$  were introduced,  $\Delta P$  is the  $P_n$  range.

$$P_n = k_1 v_n \delta_t + k_2 \sin(v_n \delta_t \Delta P) \quad (7)$$

### 4 EXPERIMENTAL PARAMETERS

For the PSO setup, known setups [1] were chosen

a) constant weights  $K=0.729$ ,  $\varphi_1 = 2.8$ ,  $\varphi_2 = 1.3$ .

b) linearly changing weight coefficients from 2 to 0.

c) constant weights  $c_1 = 2$ ,  $c_2 = 2$ .

Tests were done for all mentioned types of wall setup and all mentioned types of weights for three different ranges  $\langle -1; 1 \rangle$ ,  $\langle -10; 10 \rangle$ ,  $\langle -100; 100 \rangle$  of both  $x_1$  and  $x_2$  variables. Population consisted of 30 particles; each parameter setup has at least 400 iterations. As the fitness function, the absolute value of the function value of the tested function (Rosenbrock's, Rastigrin's, and Griewank's) was used. If the  $G_{best}$  parameter reached  $[0, 0]$  point, then simulation was stopped. Every condition setup was run 30 times to be able to have more reliable results. For the results following parameters were evaluated:  $G_{best}$  in each run and number of iterations to get to the minimum (if 400, then global was not precisely reached). The modified PSO constants were set to be  $k_1 = 0.2$ , respectively  $k_2$  was set to decrease linearly from 0.8 to zero to make the particle flight more direct (close to the classic PSO behavior).

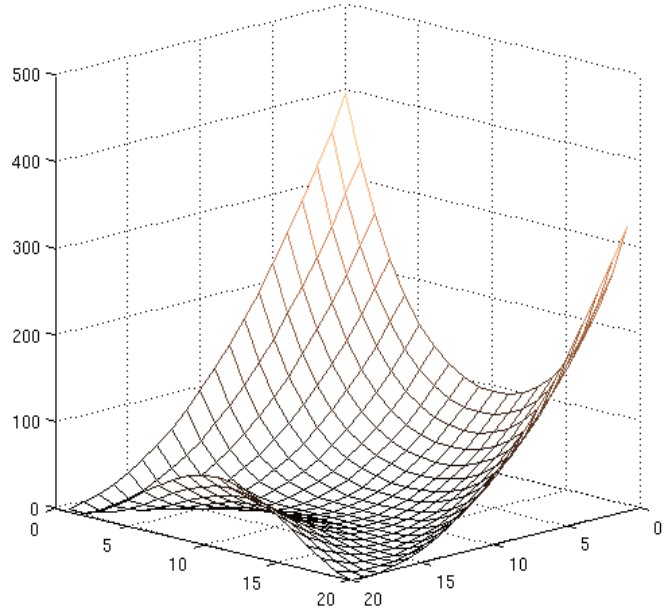


Fig 2: Rosenbrock function

## 5 DISCUSSION OF RESULTS

In the parameter range  $<-1; 1>$  all the original PSO gives perfect results for the linear weight coefficient set for Rosenbrock's function (Tab. 1), where all of the runs finished in finding the minimum. This table shows the average of the  $G_{best}$  over all runs for each condition combination. Better results for the modified version of PSO were obtained for two versions of the constant weight coefficients and absorb/invisible walls. As can be seen in the tab. 1, significantly better results were obtained for the invisible wall setup independently on the weight coefficients setup where the modified PSO was able to get more closely to the global minimum. On two higher parameter ranges, the results of the original PSO overcome the modified one especially in the case of the constant weight coefficients, where the original PSO was able to found global minimum for Rastigrin's function, for Griewank's function, it was almost in 60 % of all cases. Contrary to the  $<-1; 1>$  case, both modified and original PSO had problems to find some point closer to the minimum. The most successful were constant coefficients setup for all of the forms. In the interval of  $<-10; 10>$  the modified version of PSO had problems to find the global minimum although some of the  $G_{best}$  found were very close. This problem can be probably solved or reduced by setting up different constants ( $k_1, k_2$ ).

<i>conditions \ functions</i>	<i>Rosenbrock</i>		<i>Rastigrin</i>		<i>Griewank</i>	
<b>constant/reflect</b>	1e-5/0	2.1e-6/0	0/30	2e-3/29	0/30	0/30
<b>constant/absorb</b>	<b>1e-5/0</b>	6.1e-2/0	0.6/0	0.6/0	4e-3/0	5e-3/0
<b>constant/invisible</b>	<b>6e-4/0</b>	2e-2/0	<b>2e-3/0</b>	2/0	<b>2e-6/0</b>	1e-3/0
<b>linear/reflect</b>	0/30	0/30	2e-2/24	0/30	0/30	0/30
<b>linear/absorb</b>	0/30	0/30	0.6/0	0.5/0	1e-2/0	7e-3/0
<b>linear/invisible</b>	0/30	0/30	<b>7e-3/3</b>	0.1/0	<b>5e-7/20</b>	7e-5/0
<b>original/reflect</b>	2e-5/0	<b>2e-8/0</b>	9e-13/29	2e-3/29	0/30	0/30
<b>original/absorb</b>	4e-1/0	0.3/0	0.8/0	0.6/0	7e-3/0	7e-3/0
<b>original/invisible</b>	3e-4/0	7e-3/0	<b>5e-3/0</b>	0.1/0	<b>2e-6/0</b>	2e-4/0

*Tab 1. Average values of the global best solution over all runs/ number of particles that reached the absolute minimum for one condition setup. The results of the original PSO are on the right, modified on the left. Recognizably better results are typed bold.*

## 6 CONCLUSION

The new update formula setup was aimed to increase the PSO convergence was presented and tested. The results showed that presented modification to the original could improve the original PSO method at least for not so large variables areas. Convenient condition setups for PSO methods were presented for the original method.

## **7 FUTURE WORK**

This work can be more precisely evaluated and run statistically more important times for larger intervals of the input values and larger input parameter setup. The results could be then judged more generally with respect to the different types of tested functions. The influence of the value of constants should be worked out.

## **REFERENCES**

- [1] Robinson, J., Rahmat-Samii, Y.: Particle Swarm Optimization in Electromagnetics, IEEE Transactions on antennas and propagation, vol. 52, no 2, 2004
- [2] Mařík, V. a kol.: Umělá inteligence 3. Academia Praha, ISBN 80-200-0472-6