CHARACTERIZATION OF WAVE INTERACTION FOR NONLINEAR ELASTIC MATERIALS

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ABSTRACT

A method for ultrasonic nondestructive testing of nonlinear elastic materials (structural elements) is elaborated. Simultaneous propagation of two longitudinal waves, their reflection and interaction data are utilized. Convenient choice of the wave frequency makes it possible to analyze the data recorded on the boundaries of the specimen in terms of wave harmonic. A detailed analysis of the harmonics evolution and interaction is presented. It is shown that amplitudes of the harmonics and phase shifts are sensitive to material properties.

1 INTRODUCTION

The ultrasonic plays nowadays a prominent role in nondestructive testing (NDT). It permits the development of effective and versatile NDT method for evaluating mechanical properties of materials, for the determination of their micro- and macrostructure, flaws, inclusions, etc. Most practical applications of ultrasonic are related to solid materials. Ultrasound as a traveling wave is defined by two basic parameters: velocity and attenuation. A majority of NDT methods measure these parameters by the conventional time-of-flight method and treat the recorded data on the level of the linear theory. These methods break down then the sample is very thin, the wave velocity is frequency-dependent, and the material has complicated multiparametric properties. Two first limitations of the time-of-flight method can be eliminated by the use of Fourier transform techniques.

The basics idea is to utilize the nonlinear effects that accompany simultaneous propagation of two waves in the material. The data about simultaneous propagation, reflection, and nonlinear interaction of waves in a homogeneous nonlinear elastic material constitute the reference data for the advanced methods. The wave interaction problem is described by an analytical solution to the nonlinear wave equation. This enables as to follow the whole process of two wave nonlinear interaction in the material analytically and to analyze the evolution of the nonlinear effects in detail.

The proposed NDT method makes as of two harmonic waves excited on the opposite surfaces of the material (structural element) in terms of particles velocity and recorded on the same surfaces in terms of stress. It clears up that the appropriate choice of the wave frequency enables one to analyze the recorded data in terms of wave harmonics amplitudes and phase shifts. The recorded values of these wave characteristics are dependent on material properties. Since the analytical expressions relating wave characteristics with material properties are too cumbersome, the corresponding plots are computed and analyzed. As the result, an algorithm for nondestructive material characterization is proposed.

2 PROBLEM FORMULATION

A specimen with two parallel traction free surfaces is considered. The material of the specimen is isotropic, homogenous, and elastic, characterized by the density ρ, the Lamé coefficients λ and μ, and by the third order elastic coefficients υ_1, υ_2, and υ_3. This characterization corresponds to the five constant physically nonlinear theory of elasticity. The one-dimensional nonlinear wave propagation process in the specimen in the range of small but finite deformations is described by the equation of motion

$$[1+k_1\, U,_X(X,t)]\,U,_{XX}(X,t)-c^{-2}\, U,_{tt}(X,t)=0, \qquad (1)$$

where $U(X,t)$ denotes displacement vector. The indices after comma, X and t, indicate differentiation with respect to the Lagrangian rectangular coordinate X and time t, respectively. Equation (1) is derived using the Kirchhoff pseudostress tensor and it takes the geometrical nonlinearity of the problem into account.

The coefficients of Eq. (1), are functions of the material properties.

$$k_1=3[1+2k_0(\upsilon_1+\upsilon_2+\upsilon_3)],\quad c^2=(k_0\rho)^{-1},\quad k_0=(\lambda+2\mu)^{-1}, \qquad (2)$$

One-dimensional harmonic waves are excited in the specimen of thickness L in correspondence with the initial and boundary conditions

$$\begin{aligned} U(X,0) &= U,_t(X,0)=0,\\ U,_t(0,t) &= \varepsilon a_0 H(t)\sin\omega t,\\ U,_t(L,t) &= \varepsilon a_L H(t)\sin\omega t. \end{aligned} \qquad (3)$$

Here $H(t)$ denotes the Heaviside function, ε, a_0, and a_L are constants, and ω is the frequency. The constant ε is regarded as a small parameter $(\varepsilon << 1)$. Making use of the perturbation theory, the analytical solution to Eq. (1) is derived in the form of a series.

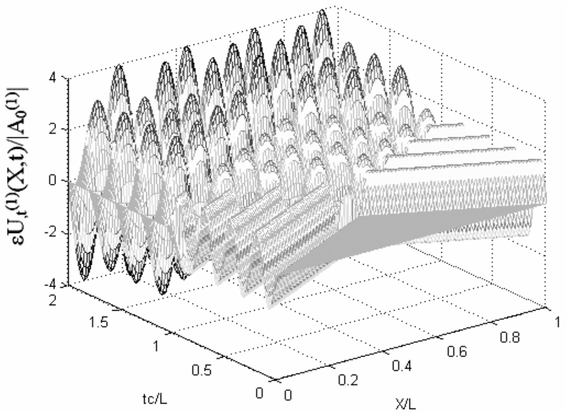

Fig. 1: *Longitudinal wave interaction in a homogeneous nonlinear elastic material,* $A_0^{(1)} \equiv \varepsilon a_0$.

$$U(X,t)=\sum_{n=1}^{\infty}\varepsilon^{n}U^{(n)}(X,t). \quad (4)$$

Solution (4) describes simultaneous propagation of the two waves in homogeneous isotropic elastic medium. The wave excited on the surface $X=0$ propagates in positive direction and wave excited on the surface $X=L$ in negative direction of the X axis (Fig. 1). Solution (4) is valid of the initial stage the distortion of the wave profile is weak and the shock wave is not generated.

3 MATERIAL CHARACTERIZATION

Investigation of wave motion in homogeneous nonlinear elastic materials is of especial importance since the obtained results may be used as reference data for elaboration of methods for characterization of materials with complicated properties. Progress in computer technology and in the development of symbolic software makes it possible to study this problem on higher level. In this paper the analytical solution derived in [2] is used. The advantage of-this approach is that all effects of wave propagation may be analyzed on the basis of analytical expressions. The effects that accompany wave propagation are dependent on the physical and geometrical properties of the material. The analytical solution analyzed in this paper enables us to derive an analytical description of the wave characteristics as a function of the material properties. This solution may be used as a theoretical basis for the following NDT methods.

1. The modified time-of-flight method. The ultrasound in the form of a harmonic wave is excited on the surface $X = 0$ of the specimen in correspondence with boundary conditions (3) where $a_L = 0$, i.e., the velocity of the material particles on the surface $X = L$ is supposed to be equal to zero. The wave process is recorded on the surface $X = L$ in terms of stress. The stress is characterized by the derivative of the displacement vector $U(X, t)$, determined by Eq. (4), with respect to the spatial coordinate X. The modification of the time-of-flight method consists in recording not only the flight time but also the distorted wave profile. The distorted wave profile is analyzed in terms of harmonics amplitudes and phase shifts. The solution (4) is used to compose the plots of the wave characteristics versus material properties. The analysis of these plots enables us to propose an algorithm of nondestructive material characterization on the basis of the recorded wave propagation data.

2. The reflected wave method. The wave process is excited on the surface $X = 0$ of the specimen in terms of particles velocity in correspondence with the boundary conditions (3) where $a_L = 0$, and it is recorded on the same surface in terms of stress. The material characterization procedure is similar to the procedure of the previous method.

3. The two waves interaction method. Two harmonic waves are excited simultaneously on two opposite parallel surfaces of the specimen in correspondence with the boundary conditions (3) in terms of particle velocity (Fig. 1). In our possession are analytical expressions for functions $U(X,t)$, $U_{,t}(X,t)$, and $U_{,X}(X, t)$ in the whole X/L, tc/L plane. Therefore, in principle, it is possible to record and analyze wave propagation and interaction in any section of the specimen. Two cases may be useful in applications.

First, the NDT of rods. Waves are excited simultaneously at both ends of the rod. The ratio of the rod diameter to the wavelength enables us to describe the wave propagation as one-dimensional. The wave motion is recorded in arbitrary rod sections in terms of $U_{,t}(X,t)$ or $U_{,X}(X, t)$. The plots of the material properties versus wave characteristics for the selected sections of the rod are computed. The analysis of these plots enables us to solve the ultrasonic material characterization problem.

Secondly, the more general case. The two wave interaction method may be applied if, there is an access at least to two parallel traction free surfaces of the specimen.

Two waves are excited simultaneously on parallel surfaces of the specimen in terms of the particle velocity (see boundary conditions (3)) and they are recorded on the same surfaces in terms of stress. The recorded data are analyzed in the time interval $0 \le tc/L < 2$ making use of the analytical expression for the function $U_{,X}(X,t)$ determined on the basis of Eq. (4). The nonlinear oscillation on the specimen boundaries may be considered as a sum of the linear constituent (first order effects) described by the first term in Eq. (4), of the second order nonlinear effects (second term in Eq. (4)), of the third order nonlinear effects, etc. It is possible to distinguish two different intervals, $0 \le tc/L < 1$ and $1 \le tc/L < 2$, for all these boundary oscillations (Fig. 2). In all cases the maximum amplification of the oscillation amplitude occurs in the wave interaction interval $1 \le tc/L < 2$. From the point of view of NDT it is essential that the inhomogeneity in material properties may be easily identified by the difference in oscillations on both boundaries. In the considered case of a homogeneous nonlinear elastic material these oscillations are theoretically identical. The profile of the oscillation and its amplitude depend on material properties. The problem is how to determine these dependences.

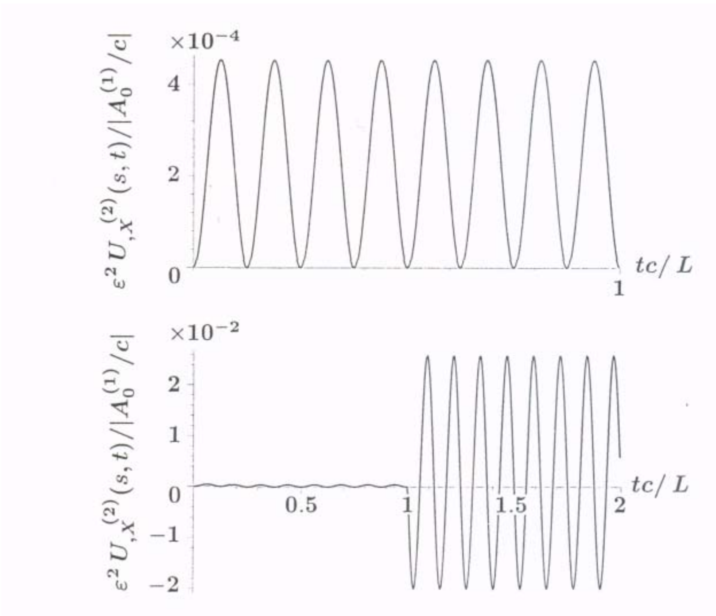

Fig. 2: *Second order nonlinear effects on boundaries.*

The analysis of the solution (4) leads to the following method for ultrasonic NDT of homogeneous nonlinear elastic materials. It can be shown that if the initial frequency of the excited waves satisfies the condition

$$\omega = 2\pi cn / L \qquad (5)$$

where n in an integer, the approach used in the modified time-of-flight method may be adapted. In this case the analytical solution (4) may be presented on the boundaries of the specimen in the form

$$U_{,X}(s,t) = A_0 + A_1 \sin(\omega\tau + \phi_1) + A_2 \sin(2\omega\tau + \phi_2) + A_3 \sin(3\omega\tau + \phi_3). \quad (6)$$

Here $\tau = t - L/c$, A_0 is the non-periodic term, amplitudes A_j and phase shifts ϕ_j have different constant values in various boundary regions. Constant s is equal to zero on the boundary $X = 0$ and to L on the boundary $X = L$.

The physical meaning of the condition (5) is that the frequency of the excited waves must be chosen so that the number of wave periods on the specimen boundary is equal to the integer in the time interval $0 \le tc/L \le 1$. The nondestructive characterization problem of a homogeneous nonlinear elastic material may be solved on the basis of Eq. (6) as follows.

The specimen of the homogeneous nonlinear elastic material is characterized, besides the dimensions, by the density ρ, by the elastic coefficients of the second order, λ and μ, and by the elastic coefficients of the third order, υ_1, υ_2 and υ_3. The peculiarity of the one-dimensional problem is that the elastic coefficients are grouped in the governing equations (1) and (2), and the elastic properties of the material may be characterized by the parameters

$$\alpha = \lambda + 2\mu, \quad \beta = 2(\upsilon_1 + \upsilon_2 + \upsilon_3). \quad (7)$$

The parameter α characterizes linear elastic properties and the parameter β nonlinear elastic properties of the material, respectively.

4 DISCUSSION AND CONCLUSIONS

The topic of this paper may be considered as a part of the project to elaborate a relatively simple method for NDT of materials (structural elements) with complicated properties. The presented front patterns of wave components characterize the nature of the nonlinear wave process in the homogeneous material and these patterns may become a good reference data to distinguish materials with more complicated properties.

Three different possibilities to use the considered nonlinear wave propagation data in NDT of materials are proposed. This method may be efficient by NDT of inhomogeneous materials. It is noticed that convenient choice of the excitation frequency enables one to transform the description of the wave process on the boundaries into the form of harmonics. The dependence of harmonics amplitudes and phase shifts on the material properties is analysed on the basis of corresponding plots. An algorithm for nondestructive characterization of homogeneous nonlinear elastic materials is proposed.

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