# ANALYSIS OF STIRLING ENGINE CHARACTERISTICS BY SCHMIDT'S THEORY

Ing. Jan MACHÁČEK, Doctoral Degree Programme (2)
Dept. of Electrical Power Engineering, FEEC, BUT
e-mail: xmacha08@stud.feec.vutbr.cz

Supervised by: Dr. Jan Gregor

**ABSTRACT**

This article deals with problems associated with its operating characteristics. The Stirling engine's parameters depend on both theoretical analysis applying Schmidt's Theory, which mathematically describes the Stirling cycle, and on experimental measurements, which reveal its actual properties. The design and contraction of the Stirling engine should be based on results of theoretical calculations and applied physics as well as data derived from practical measurements.

## 1 ANALYSIS OF CHARACTERISTICS BY SCHMIDT'S THEORY

In 1861, E. Schmidt carried out mathematical modeling and Stirling engine analysis. His theory is based on harmonic pistons and device nodes movements, ideal isothermal expansion and compression and ideal regeneration. This method is considerably closer to realization of Stirling engine, even though some idealization of process is included, than purely theoretical approach resulting from Stirling cycle. Schmidt's theory assumptions:

- Processes of regeneration are ideal; there is a perfect regeneration.
- Instant value of system pressure is constant.
- Conditions of the working gas is changed as an ideal gas.
- Gas volume changes are sinusoidal.
- Temperature of cylinder walls and piston is constant.
- Temperature of used material/gas is constant.
- Device revolves at the constant rate.
- The expansion process and the compression process changes isothermal.
- The regenerator gas temperature is an average of the expansion gas temperature and the compression gas temperature.
- The amount of working gas is constant.

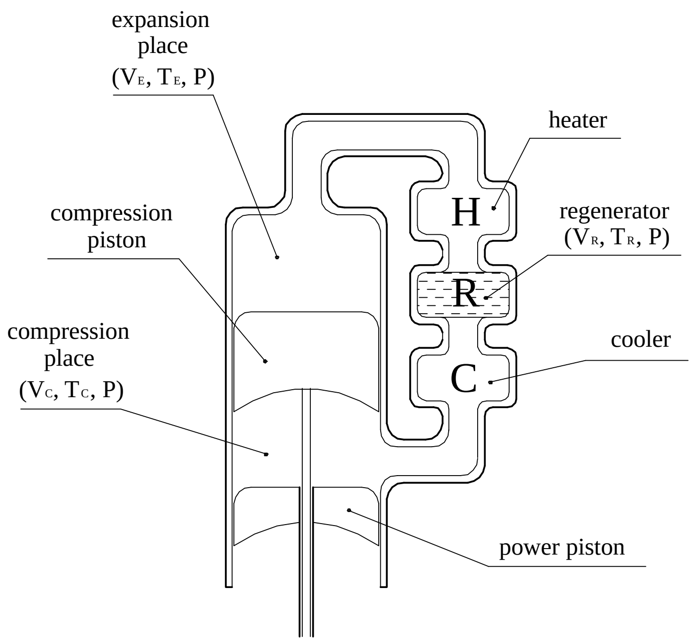


Figure 1: Construction of β engine modification.

The expansion momental volume - $V_E$ and the compression momental volume - $V_C$ are described in the following equations, with a swept volume of a displacer piston - $V_{SE}$, a swept volume of a power piston - $V_{SC}$ and a phase angle -dx between the displacer piston and power piston.

$$V_E = \frac{V_{SE}}{2} \cdot (1 - \cos x) + V_{DE} \tag{1}$$

$$V_C = \frac{V_{SE}}{2} \cdot (1 + \cos x) + \frac{V_{SC}}{2} \cdot [1 + \cos(x - \varphi)] + V_{DC} - V_B \tag{2}$$

In the case of the Beta-type Stirling engine, the displacer piston and the power piston are located in the same cylinder. When both pistons overlaps there stroke, an effective working space is created. The overlap volume - $V_B$ in equation (2) can be calculated in the next equation:

$$V_B = \frac{V_{SE} + V_{SC}}{2} - \sqrt{\frac{V_{SE}^2 + V_{SC}^2}{4} - \frac{V_{SE} \cdot V_{SC}}{2} \cdot \cos\varphi} \tag{3}$$

Then the total momental volume - $V$ is found in equation (4):

$$V = V_C + V_R + V_E \tag{4}$$

The engine pressure - P based the mean pressure - $P_{mean}$, the minimum pressure - $P_{min}$ and the maximum pressure - $P_{max}$ are described in the following equations like the Alpha-type Stirling engine.

$$p = \frac{p_{mean} \cdot \sqrt{1 - c^2}}{1 - c \cdot \cos(x - \psi)} = \frac{p_{min} \cdot (1 + c)}{1 - c \cdot \cos(x - \psi)} = \frac{p_{max} \cdot (1 - c)}{1 - c \cdot \cos(x - \psi)} \tag{5}$$

Several ratios and coefficients are defined as follows:

$$\tau = \frac{T_C}{T_E} \tag{6}$$

$$\nu = \frac{V_{SC}}{V_{SE}} \tag{7}$$

$$X_B = \frac{V_B}{V_{SE}} \tag{8}$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \tag{9}$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \tag{10}$$

$$X_R = \frac{V_R}{V_{SE}} \tag{11}$$

$$a = \sin^{-1} \frac{\nu \cdot \sin\varphi}{t + \cos\varphi + 1} \tag{12}$$

$$\psi = \tan^{-1} \frac{\nu \cdot \sin\varphi}{\sqrt{\nu^2 + \lambda^2 + 2 \cdot \nu \cdot \lambda \cdot \cos\varphi}} \tag{13}$$

$$\lambda = \tau - 1 \tag{14}$$

$$S = \tau + 2 \cdot \tau \cdot X_{DE} + \frac{4 \cdot \tau \cdot X_R}{1 + \tau} + \nu + 2 \cdot X_{DC} + 1 - 2 \cdot X_B \tag{15}$$

$$B = \sqrt{\tau^2 + 2 \cdot (\tau - 1) \cdot \nu \cdot \cos\varphi + \nu^2 - 2 \cdot \tau + 1} \tag{16}$$

$$c = \frac{B}{S} \tag{17}$$

Amount of heat accepted by gas is:

$$Q_E = \frac{p_{mean} \cdot V_{SE} \cdot \pi \cdot c \cdot \sin\psi}{1 + \sqrt{1 - c^2}} \tag{18}$$

Amount of transferred heat is:

$$Q_C = -Q_E \cdot \tau \tag{19}$$

Total power performance $W$ are.

$$W = Q_E + Q_C \tag{20}$$

## 1.1 CALCULATED AND MEASURED CHARACTERISTICS

We used real constructional parameters of device to calculate coefficients and values calculations described by Schmidt's theory. This is the only way how it is possible objectively compare features of real and simulated engine. It is true some characteristics of theoretic analyze are not applicable in practice. It is concerned as for mathematic description of motor piston move that is differ from theoretic projection of this move. Input data of model:

- diameter of cylinder and piston $= 0.072\ m$,
- diameter of displacer piston $= 0.168\ m$,
- diameter of move piston cylinder $= 0.17\ m$,
- stroke of piston $= 0.036\ m$,
- temperature of expansion $T_E = 155\ °C$,
- temperature of compression $T_C = 20\ °C$,
- phase angle $\varphi = \frac{\pi}{2}$,
- presure $p_{str} = 101325\ Pa$,
- stroke capacity of expansion space $V_{SE} = 0.0008171\ m^3$,
- stroke capacity of compression space $V_{SC} = 0.0001465\ m^3$,
- dead volume of expansion space $V_{DE} = 0.0004478\ m^3$,
- dead volume of expansion space from all compression space $V_{DC} = 0.00000384\ m^3$,
- motor speed $n = 70\ min^{-1}$.

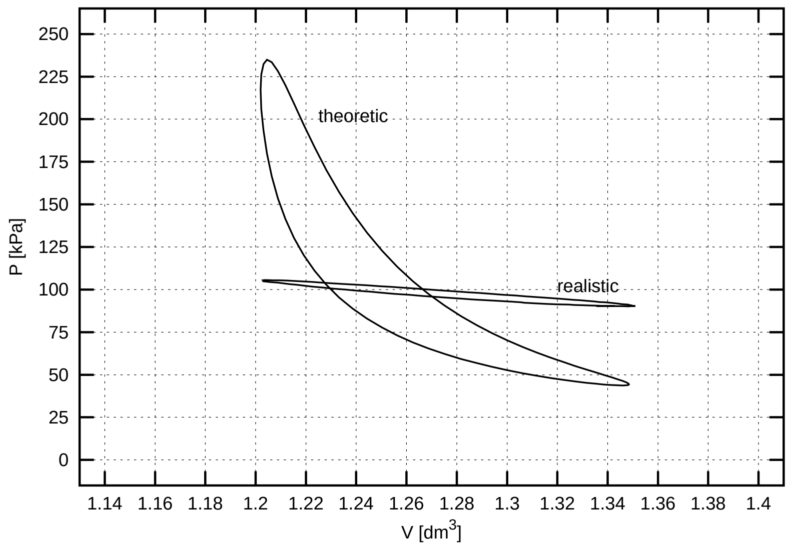


Figure 2: p - V diagrams of theoretic and realistic engine.

Expansion temperature, compression temperature and motor speed we gained by measure with temperature input $P = 130W$. The other parameters are related with motor construction and are constant. We used Schmidt beta-type theory for our analyze because it is most alike to model constructed in laboratory UEEN. Constitute to these theory formulas we gained theoretic $p - V$ diagram demonstrated on Figure 2. It is related to elected temperatures and motor speed. Theoretic $p - V$ diagram we compared with real diagram gained by measure. Diametrical difference between theoretic and real diagram is evident. So our conclusion is follows: expansion and compression processes do not run by these measure temperatures. The reason is we are not able to measure gas temperature by expansion and compression. Temperatures that we are measure are temperatures of hot and cold motor board. Real gas temperatures are so wholly different. That is way we did new calculation and the temperatures adapted in order to theoretic and real diagram are equivalent. We found out that the temperatures proportion for theoretic and real diagram equivalent is cca 0.843 and it could be $T_E = 35.0\ ^{\circ}C$, $T_C = 29.5\ ^{\circ}C$.

## 2 CONCLUSION

In this article is described analyze of Stirling engine characteristics by Schmidt theory. We found out that Stirling engine has many imperfections. Real expansion and compression temperature is very different from measured values which ones we considered close to compression and expansion temperatures. This difference is caused by missing of regenerator function. Construction and used material of regenerator are unsuitable and that is way optimization and construction of new regenerator is main object of our next research.


## ACKNOWLEDGMENT

This paper contains the results of research works funded from project No. MSM 0021630516 of the Ministry of Education, Youth and Sports of the Czech Republic.


## REFERENCES


[1] Haywood, R. W.: Analysis of Engineering cycle. Pergamon Press, fourth edition, 1991.

[2] Macháček, J. Gregor, J.: Provozní vlastnosti modelu Stirlingova motoru. Brno: Sborník konference Electric power engineering. 2004. 110 s. ISBN 80-214-2642-X

[3] Macháček, J. Gregor, J.: Model of Stirling engine. Brno: Proceendings of the 10th conference Student EEICT 2004 volume 3. 696 s. ISBN 80-214-2636-5

[4] Walker, G.: Mašiny, rabotajuščie po ciklu Stirlinga. Moskva: Energija. 1978. TK-0198.998

[5] To Bekkoame Home Page, Stirling engine home page [online]. © 1995, [cit. 2004-07-25]. Dostupné z: ⟨http://www.bekkoame.ne.jp/∼khirata⟩. Schmidt theory for Stirling engines.