

# MODELING AND CONTROL OF NONLINEARITIES HYSTERESIS TYPE

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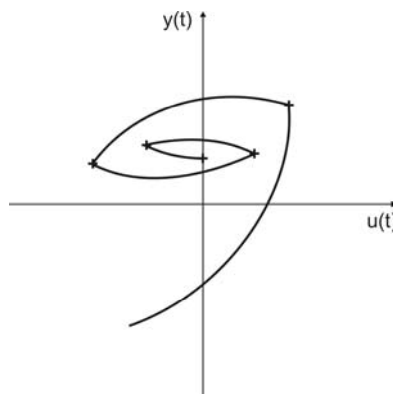
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## ABSTRACT

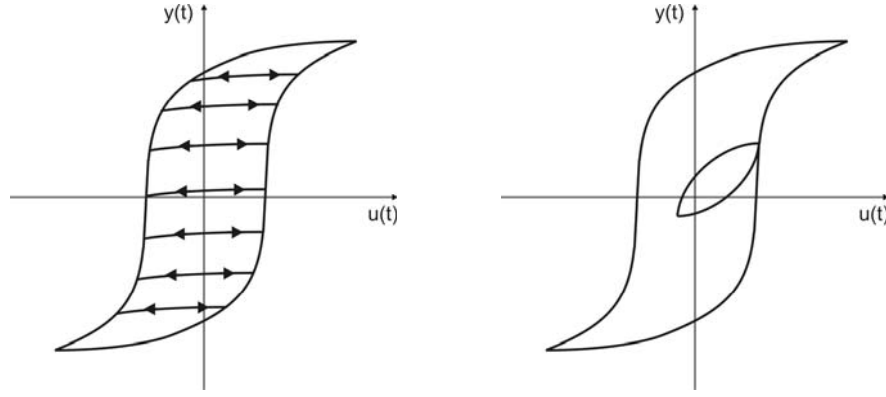
This paper discusses the issue of nonlinearity hysteresis type. At the present this problem is often solved in association with untraditional actuators used in robotics. An actuator based on SMA (Shape Memory Alloy) principle can be chosen as an epitome of this problem. The ways to model varied types of hysteresis are described in this paper. Moreover, the advantages and disadvantages of described approaches are considered and some results of the experiments which have been made with particular models are presented.

## 1 INTRODUCTION

The problem of hysteresis has been known for a long time. This phenomenon can be found in many fields such as economy, mechanics, electroengineering, chemistry etc. The hysteresis is typically described as a hysteresis loop (fig.2). The behaviour which is called hysteresis is a result of nonlinearity multilayer branching (fig.1).



**Fig. 1:** *Multilayer nonlinearity.*



**Fig. 2:** *Hysteresis loop a)classical hysteresis (LM) b)hysteresis with internal loop(NM).*

Two types of hysteresis can be found: firstly hysteresis with local memory(LM) and secondly, hysteresis with non-local memory(NM). The figure 2a shows behaviour of system with local-memory. This curve is usually called classical hysteresis. There is a vast number of models which describe this behaviour. The complexity of each model depends on quality of modeling of exact problem. In other words, it's given by degree of linearization. LM-hysteresis can be simply represented by relay or backlash operator. The models of these simple nonlinearities are very often presented in libraries of software which are commonly used for modeling (MATLAB, ANSYS, etc.) or the model of the LM-hysteresis can be built by basic functions (sin, cos, exp etc.).

There is the second type of hysteresis also. The figure 2b shows a NM-hysteresis. The main difference between the NM-hysteresis and LM-hysteresis is in "internal loops" which are usually possible to find in behaviour of systems described by this phenomenon. It's easy to prove that the LM-hysteresis is a limitative state of NL-hysteresis. It's obvious that the modeling of hysteresis with internal loops is more complicated than modeling of LM-hysteresis. There are couple of methods which are used for the modeling of hysteresis with internal loops. Two of them will be described below.

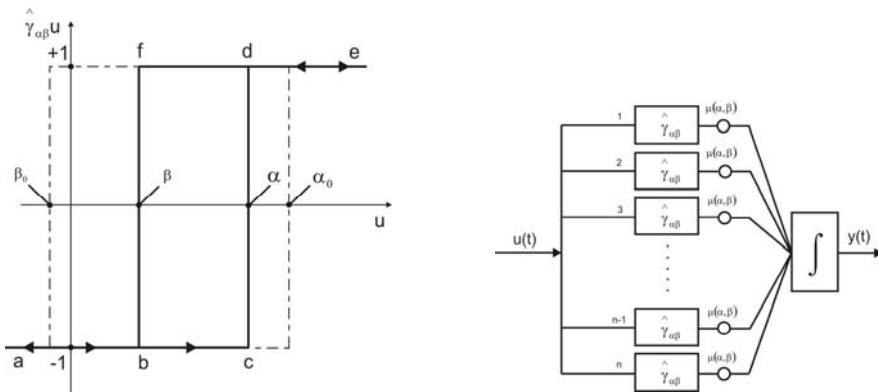
## 2 PREISACH'S APPROACH

Ferenc Preisach was designed this method in first half of twentieth century. The Preisach's approach has been inspired by physical phenomenon of magnetization. The Preisach's model is based on a principle of partial nonlinearities summation defined below. The figure 3b shows a structure of this model. The basic nonlinearity for this model is nonlinearity type relay (fig. 3a). These relays can assumed values of +1 or 1. The output of the whole model is given by summation of all defined simple nonlinearities multiplied by special weight function called the Preisach's operator. This principle can be mathematically noted as follows:

$$f(t) = \hat{\Gamma} u(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha, \beta} u(t) d\alpha d\beta \quad (2.1)$$

where  $\mu(\alpha, \beta)$  is Preisach's operator and  $\hat{\gamma}_{\alpha, \beta}$  is operator described the simple nonlinearity mentioned above. The Preisach's operator is given by history of input function

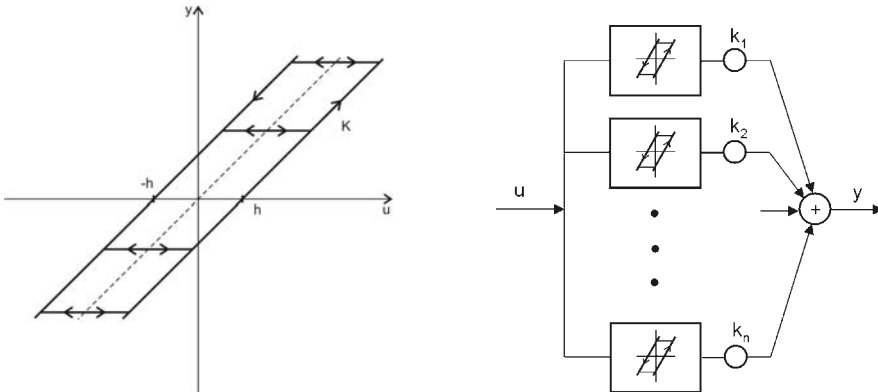
minimum and maximum at the past. It's clear that the precision of this model depends on the number of partial nonlinearities implemented inside the model.



**Fig. 3:** Preisach's model a)partial nonlinearity b)structure of model.

### 3 PRANDTL – ISHLINSKII'S MODEL

This model is based on a same principle as a Praisach's model, where the idea was to sum the partial nonlinearities (fig. 4b). This conception is also called „main block“. The nonlinearity type backlash is used as an basic block in P-I model (see fig. 4a).



**Fig. 4:** Prandtl-Ishlinski model a)backlash operator b)structure of model.

In this case is implemented the weight function as well. The output of this model is given by summing of partial nonlinearities multiplied be defined weight function. This function has been mentioned above. The model was improved by authors later. They changed the partial nonlinearity type backlash with saturation instead backlash type nonlinearity.

The output from model without the saturation can be note as follows:

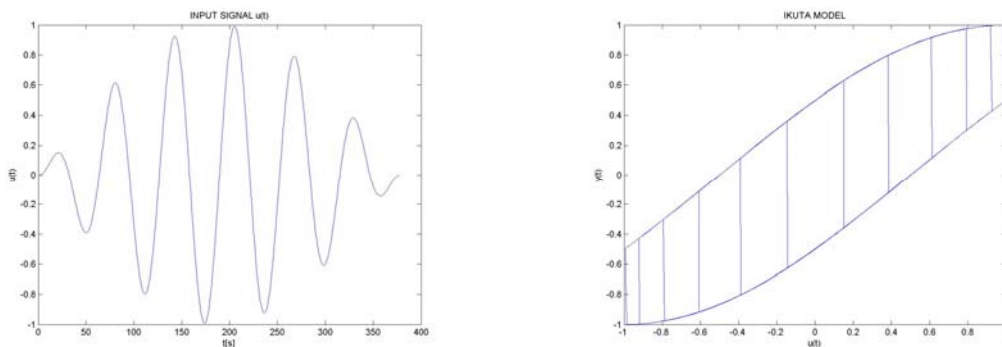
$$\begin{aligned}
y(k) &= K(u(k) + h) & \text{for } u(k) < u_- \\
y(k) &= y(k+1) & \text{for } u(k) \in \langle u_-, u_+ \rangle \\
y(k) &= K(u(k) - h) & \text{for } u(k) > u_+
\end{aligned} \tag{3.1}$$

where

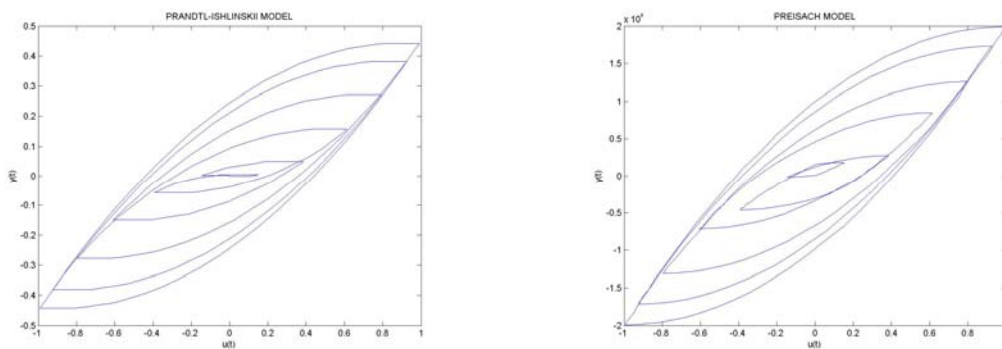
$$u_- = \frac{y(k-1)}{K} - h, \quad u_+ = \frac{y(k-1)}{K} + h \tag{3.2}$$

#### 4 SIMULATIONS

The simulations with above described models have been made. See the following charts for the comparison. There is the input function which has been used for all models and output signals from three types of models. Ikuta's model has been added for the better comparison with Preisach's and Prandtl-Ishlinskii. This model is use for modeling of hysteresis as well. The main Ikuta's model disadvantage is a disability to model the hysteresis with internal loops. Ikuta's model is based on sinus function.



**Fig. 5:** a) input function b) Ikuta's model.



**Fig. 6:** a) Prandtl-Ishlinskii's model b) Preisach's model.

The Preisach's model has been created by 231 simple elements type relay. The Prandtl-Ishlinskii model has been created by 8 elements type no-saturated backlash. The weight functions weren't implemented.

## **5 CONCLUSION**

This article presented two methods of NM-hysteresis modeling. The methods presented above are more complicated than usually used ways of LM-hysteresis modeling. The advantages of mentioned approaches have been shown. The Preisach's and Prandtl-Ishlinskii models are quite arduous to computing, both of them is possible to tune for particular application of course. It's clear that for the quality control the correct implementation of these models into the controlling circuit is necessary. The methods of controlling with model have been described in many publications.

## **ACKNOWLEDGEMENTS**

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