BASICS OF COMPUTER AIDED CIRCUITS ANALYSIS

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ABSTRACT

This article deals with basic issues which we can meet when we want to analyze switched circuits with nonlinear models of elements using PC. Theoretical basements and their possible solutions as computer algorithms for nonlinear switched circuit analysis are discussed in the text below.

1 INTRODUCTION

During nonlinear dynamic circuit analysis (analog or switched) we have to solve a lot of issues. First we have to choose a good method how to effectively (using PC) analyze wide sort of nonlinear circuits. Modified Nodal Analysis seems to be the best choice. It allows us to analyze circuit with voltage and current defined elements and simply compute any other circuit variables (not only nodal voltages). Second it is necessary to implement an algorithm for non-linear equations solving (when analyzing nonlinear circuits) and third we have to solve differential equations also we must choose effective numerical integration method. In some cases we use charges and fluxes as circuit variables when we are analyzing nonlinear dynamical circuit in time domain because of some reasons.

Analysis of switched circuits (also dynamic circuits) brings other issues we have to study. There is necessary to use charges and fluxes as circuit variables in analysis methods because these variables acquire finite values in switching times (instead currents and voltages).

2 MODIFIED NODAL ANALYSIS (MNA)

Modified nodal analysis (MNA) is general and often used method for circuit analysis using computer. You can see its general definition [2] in nonlinear differential equations system:

$$\mathbf{A}_{\mathrm{I}}\mathbf{i}_{\mathrm{I}} + \mathbf{A}_{\mathrm{II}}\mathbf{i}_{\mathrm{II}} = \mathbf{0}\,, \tag{1}$$

$$\mathbf{i}_{\mathrm{I}} = \mathbf{f}_{\mathrm{I}}\left(\mathbf{A}^{\mathrm{t}}\mathbf{v}_n, \frac{d\mathbf{q}_{\mathrm{I}}\left(\mathbf{A}^{\mathrm{t}}\mathbf{v}_n\right)}{dt}, \mathbf{i}_{\mathrm{II}}, \frac{d\boldsymbol{\psi}_{\mathrm{I}}\left(\mathbf{i}_{\mathrm{II}}\right)}{dx}, \mathbf{s}(t), t \right), \tag{2}$$

$$\mathbf{f}_{\mathrm{II}}\left(\mathbf{A}^{\mathrm{t}}\mathbf{v}_n, \frac{d\mathbf{q}_{\mathrm{II}}\left(\mathbf{A}^{\mathrm{t}}\mathbf{v}_n\right)}{dt}, \mathbf{i}_{\mathrm{II}}, \frac{d\boldsymbol{\psi}_{\mathrm{II}}\left(\mathbf{i}_{\mathrm{II}}\right)}{dt}, \mathbf{s}(t), t \right) = \mathbf{0}, \tag{3}$$

where **A** is an incidence matrix, $\mathbf{v}_n$ is a vector of unknowns nodal voltages, **s** is vector of excitations, variables with index I are variables of current-defined branch (defined by equation $i = f\left(\mathbf{v}, \mathbf{i}_{\mathrm{II}}\right)$) and variables with index II are variables of voltage-defined branch and branch currents, which are to be considered in analysis (we want to compute them). Equation set (1) – (3) is used for analysis of wide sort of nonlinear dynamic circuits. When analyzed using computer we finally solve a system of linear equations $\mathbf{G} \cdot \mathbf{x} = \mathbf{y}$, where matrix **G** characterize the circuit, **x** is a vector of unknowns and **y** a is vector of excitations. The matrix **G** describes how elements are interconnected and how they are defined. Using computer we can easily set up this matrix using **stamps**. We obtain these stamps for each element from structure containing a model of an element. The matrix **G** is also a sum of stamps of all elements.

3 COMPUTER AIDED CIRCUIT ANALYSIS

3.1 LINEAR STATIC CIRCUITS

This is the simplest kind of circuits. When we want to understand the principles of circuit analysis and to form algorithms for computer aided circuit analysis, first we have to begin with these types of circuit with rather demonstration importance. MNA equations change as follows

$$\begin{bmatrix} \mathbf{A}_{\mathrm{I}}\mathbf{G}_{\mathrm{I}}\mathbf{A}^{\mathrm{t}} & \mathbf{A}_{\mathrm{I}}\mathbf{K}_{\mathrm{I}} + \mathbf{A}_{\mathrm{II}} \\ \mathbf{L}_{\mathrm{II}}\mathbf{A}^{\mathrm{t}} & \mathbf{K}_{\mathrm{II}} \end{bmatrix} \begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{\mathrm{I}}\mathbf{I}_{\mathrm{I}} \\ \mathbf{s}_{\mathrm{II}} \end{bmatrix}, \tag{4}$$

where **G** is conductance matrix, **K** and **L** are coefficients of controlled current and voltage sources in each branch and **I**, **s** are excitations vectors. LHS (Left Hand Side) matrix is sum of stamps as mentioned above. View fig. 1 and see tab. 1 for an example how to set up LHS matrix using stamps. We obtain these stamps from models of circuit elements. Models are actually functions. Their input variables are model parameters (e.g. for conductor it is its conductance), output of these functions are stamps.

	1	2
1	G	-G
2	-G	G

	1	2	i_c
1			β
2			-β

	1	2	i_e	
1			1	
2			-1	
i_e	1	-1		U

	1	2	3	4	i_e
1					1
2					-1
i_e	1	-1	-g	g	

Tab. 1: *Stamps of (from left) conductor G, current-controlled current source, voltage source and voltage-controlled voltage source. Empty cells mean zero elements. Double line isolates excitation RHS vector.*

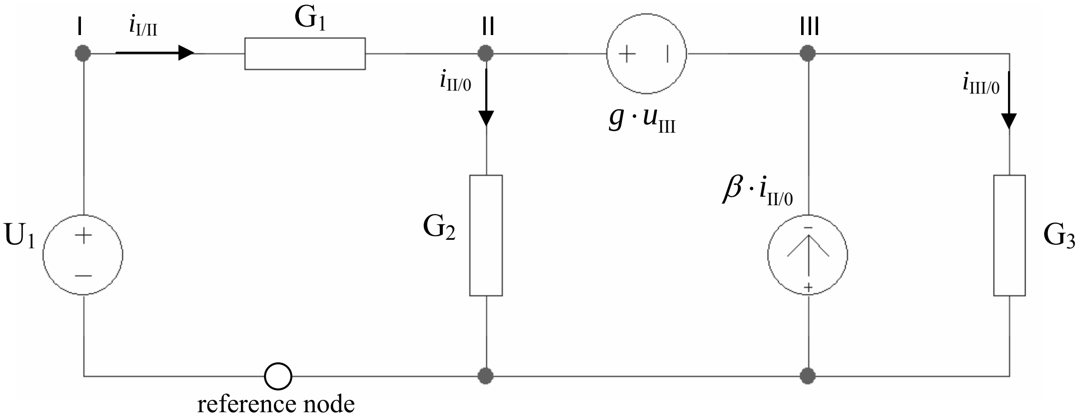

Fig. 1: *An example of a linear circuit. Some variables are not drawn.*

The current i_e means the current through an element and the current i_c means the control current. Numbers of poles in stamps correspond to the numbers of nodes, which given element is connected to. Columns and rows with currents are added into LHS matrix – see Tab. 2:.

Computer algorithm consists of few steps [1]. First, the data structure describing entire circuit must be loaded and analyzed. For each element its stamp must be obtained and added into LHS matrix. After this cycle, system of linear equations must be solved. Gaussian elimination method is used in Matlab environment. We can simply solve the system of linear equations $\mathbf{G} \cdot \mathbf{x} = \mathbf{y}$ using the following Matlab command:

$$\mathbf{x} = \mathbf{G} \backslash \mathbf{y}; \tag{5}$$

	1	2	3	$i_{I/0}$	$i_{II/0}$	$i_{III/0}$	$i_{II/III}$	
1	G_1	$-G_1$		1				
2	$-G_1$	G_1			1		1	
3					$-\beta$	1	-1	
$i_{I/0}$	1							U_1
$i_{II/0}$		G_2			-1			
$i_{III/0}$			G_3			-1		
$i_{II/III}$		1	-g-1					

Tab. 2: *Final LHS matrix for circuit on Fig. 1:.*

3.2 NONLINEAR STATIC CIRCUITS

We solve this type of circuits during d.c. analysis (direct current), where capacitors are replaced by open circuit and inductors by short circuit. We can use following steps during AC analysis, but excitations must vary in time very slowly. Also this is useful during DC sweep analysis. We change MNA equations (1) – (3) with respect to this type of circuits by zeroing all derivatives, see eq. (6) – (8).

$$\mathbf{A}_\mathrm{I}\mathbf{i}_\mathrm{I} + \mathbf{A}_\mathrm{II}\mathbf{i}_\mathrm{II} = \mathbf{0}, \tag{6}$$

$$\mathbf{i}_\mathrm{I} = \mathbf{f}_\mathrm{I}\left(\mathbf{A}^\mathrm{t}\mathbf{v}_n, \mathbf{0}, \mathbf{i}_\mathrm{II}, \mathbf{0}, \mathbf{s}(t), t\right), \tag{7}$$

$$\mathbf{f}_\mathrm{II}\left(\mathbf{A}^\mathrm{t}\mathbf{v}_n, \mathbf{0}, \mathbf{i}_\mathrm{II}, \mathbf{0}, \mathbf{s}(t), t\right). \tag{8}$$

System of equations (6) - (8) is generally nonlinear, also we have to use some numerical method to solve this using a computer. Newton-Raphson's method is widely used for solution of nonlinear equations in form $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. The principle of the method is defined by eq. (9).

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \left[\mathbf{f}'\left(\mathbf{x}^{(k)}\right)\right]^{-1} \cdot \mathbf{f}\left(\mathbf{x}^{(k)}\right), \tag{9}$$

where k is number of the current iteration cycle. For better computational efficiency we write eq. (8) in form of incremental linearization equation (10) – (11).

$$\mathbf{f}'\left(\mathbf{x}^{(k)}\right) \cdot \Delta\mathbf{x}^{(k+1)} = -\mathbf{f}\left(\mathbf{x}^{(k)}\right), \tag{10}$$

$$\Delta\mathbf{x}^{(k+1)} = \mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}. \tag{11}$$

These iterations may not converge. Whether the method converges it depends on $\mathbf{f}(\mathbf{x})$ waveform and initial point $\mathbf{x}^{(0)}$ selection. We use some step limiting algorithm to speed up iterations or to enlarge reliability of convergence (e.g. $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k+1)} \cdot \Delta\mathbf{x}^{(k+1)}$, where $\alpha^{(k+1)}$ is limiting factor). We can use incremental linearization for stamp creation of a nonlinear element. An example of stamp of incremental linearized static model of a diode can be seen in tab. 3. Diode is defined by equation $i_\mathrm{d} = I_\mathrm{s}\left(e^{\Theta u_\mathrm{d}} - 1\right)$.

	a	k	
a	$i_\mathrm{d}'\left(u^{(k)}\right)$	$-i_\mathrm{d}'\left(u^{(k)}\right)$	$-i_\mathrm{d}\left(u^{(k)}\right)$
k	$-i_\mathrm{d}'\left(u^{(k)}\right)$	$i_\mathrm{d}'\left(u^{(k)}\right)$	$i_\mathrm{d}\left(u^{(k)}\right)$

Tab. 3: *Stamp of incremental linearized static model of a diode for MNA.*

3.3 NONLINEAR DYNAMIC CIRCUITS

For description of this type of circuits we use eqs. (1) - (3). These equations are used for transient analysis. But how we can see, we must deal with a system of nonlinear differential equations. First logical step is to eliminate derivatives in this system. We use numerical integration methods, so we get system of nonlinear equations (solution method for this type of equation system has already been described above). Numerical integration means dividing continuous time scale into sequence of discrete points $\{t_n\}$. Neighbor point distance is called integration step $h = t_{n+1} - t_n$. We then usually get system of difference equations in form

$$x_{n+1} = \mathcal{F}_n\left(x_n, x_{n-1}, \ldots, x_{n+1-p}, \dot{x}_n, \dot{x}_{n-1}, \ldots, \dot{x}_{n+1-p}\right). \tag{12}$$

Backward Differentiation Formulae (BDF) of *l*-th order, expressed by general formula (13), is integration method often used.

$$\dot{x}_n = \sum_{k=0}^{l} \gamma_k^{(l)} x_{n-k} \ . \qquad (13)$$

As discussed in [2] there is better to use charges-fluxes due to the reason of solution accuracy and stability. E.g. general nonlinear charge-defined branch $i = \dot{q}(u)$ has after discretization and incremental linearization following form

$$\Delta i_{n+1}^{(k+1)} = \gamma \Delta q_{n+1}^{(k+1)} + \Delta k_{n+1}^{(k)} \qquad \Delta q_{n+1}^{(k+1)} = C\left(u_{n+1}^{(k)}\right) \Delta u_{n+1}^{(k+1)} + \Delta j_{n+1}^{(k)} . \qquad (14)$$

If we use voltage-defined expression $i = C(u)\dot{u}$, we get formulae with derivatives of capacity. Discretized and incremental linearized models of elements (stamps) can be formed.

3.4 SWITCHED CIRCUITS

Switched circuits are special cases of nonlinear dynamical circuits. There are elements such as ideal switch (we can also form its stamp). Algorithm is very similar to the algorithms for nonlinear dynamic circuit analysis with some differences [4]. E.g. in switching time $t = t_i$ using the values of circuit variables close before $t = t_{i^-}$ we set initial values of solution close after time $t = t_{i^+}$.

4 CONCLUSION

We must weight the options of increasing robustness of used iteration methods (enlarge reliability of convergence and solvability). In MNA method we must strike wide sort of special types and configurations of circuits, e.g. isolated nodes. We are being created computer algorithms of circuit analysis approaches presented here in mathematical tool Matlab v.14. Set of created functions form a simulation core of more complex simulation program called SCISIP [3].

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