WALSH FUNCTIONS – THE TUTORIAL PROGRAM

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ABSTRACT

The article deals with program, which is able to generate Walsh functions and to demonstrate binary data spreading for code division multiple access purpose. Walsh functions are generated using Rademacher functions. This way of generating of the Walsh functions is also described in this article. Program is designated for laboratory exercise and for tuition. Program is completely created in Matlab.

1 INTRODUCTION

At present the CDMA (Code Division Multiple Access) is the main principle of the wireless communications. In this way the spreading codes are mentioned. There are two basic groups of spreading codes. The first group is formed by pseudo noise codes. In the second group are orthogonal codes. The orthogonal codes are just used for code division (channelizing) of channels operated on the same frequency and in the same time.

There are several codes, which satisfy the orthogonality condition. One possibility forms Walsh functions. Walsh functions of order N are the set of functions of length N. This set is formed by N/n subsets. In reality only one function from the subset can be used. The value of n depends on length of data sequence, which is coded by Walsh function. The reason for this restriction is the orthogonality of the spreaded bit stream, which must be satisfied too.

2 WALSH FUNCTION GENERATION

This program generates Walsh functions and Walsh sequences using Rademacher functions. This process requires Rademacher functions generation (described in [1]) at first. Rademacher functions of order N are set of $1+\log_2 N$ orthogonal functions consisting of $N = 2^K$ rectangular pulses. These pulses assume alternately the values +1 and -1 in an interval of $(0, T)$. The Rademacher functions of order N are defined by the relation

$$R_n(t) = \operatorname{sgn}(\sin 2^n \pi t), \qquad t \in (0,T), \quad n = 1,2,..,K \tag{1}$$

where $R_0(t)=1$ and

$$\operatorname{sgn}(x) \equiv \begin{Bmatrix} -1 & for & x < 0 \\ 0 & for & x = 0 \\ 1 & for & x > 0 \end{Bmatrix} \quad (2)$$

The Rademacher Functions are constructed by implementation of definition relation into the program. After generating Rademacher functions are these converted to Rademacher sequences. This operation yields conversion into binary logic (+1→0 and -1→1).

The Walsh sequences are generated as a modulo-2 sum of Rademacher functions. This process is based on Walsh function index sequence conversion into the Gray code. Index sequence of Walsh function is simply Walsh function index in binary code $X_i = (x_{i1}, x_{i2}, \ldots, x_{iK})$. The Gray code $G_i = (g_{i1}, g_{i2}, \ldots, g_{iK})$ is created as follows:

$$g_{i1} = x_{i1} \quad (3a)$$

$$g_{ij} = x_{i,j-1} \oplus x_{ij}, \qquad j = 2,3,...,K \quad (3b)$$

The Walsh sequences are exactly formed as the modulo-2 sum of R_0 and the Rademacher sequences $\{R_j\}$ that are not associated with zero values of G_i. This can be formulated as follows:

$$W_i = R_0 \oplus \sum_{j:g_{ij}=1} R_{K+1-j} \quad (4a)$$

$$W_i = R_0 \oplus [R_K \quad if \quad g_{i1} = 1] \oplus [R_{K-1} \quad if \quad g_{i2} = 1] \oplus \ldots \oplus [R_1 \quad if \quad g_{iK} = 1] \quad (4b)$$

The result of this operation is formed as set of N Walsh sequences of order N.

The Walsh functions can be formed from sequences using reverse conversion which is mentioned above. Another way of Walsh functions forming is described in [2]. Walsh functions are formed directly as the product of $R_0(t)$ and the Rademacher functions $\{R_j(t)\}$ that are not associated with zero values of G_i. This can be formulated as follows:

$$W_i(t) = R_0(t) \times \prod_{j:g_{ij}=1} R_{K+1-j}(t) \quad (5a)$$

$$W(t)_i = R_0(t) \times [R_K(t) \quad if \quad g_{i1} = 1] \times [R_{K-1}(t) \quad if \quad g_{i2} = 1] \times \ldots \times [R_1(t) \quad if \quad g_{iK} = 1] \quad (5b)$$

Both of showed ways of Walsh functions generation are quite similar and using one of them depends on the way of consequent use.

Described program uses both Walsh functions and Walsh sequences. Therefore the Walsh sequences are generated. The program includes the functions that can convert sequences to functions and functions to sequences.

3 PROGRAM PREFERENCES

This program is designed as tutorial for laboratory exercise. The application window is shown in Fig. 1. The program consists of three main parts:

- Walsh sequences generating function and sequences ↔ functions converting functions
- Functions for demonstration spreading process, code division, Walsh functions orthogonality and signal reconstruction process

- User interface with controls for drawing figures with demonstration pictures

Walsh sequences generator is a function, which generates Walsh sequences of order from 2 to 128. Upper limit is because good picture displaying transparency purpose. Conversion functions serves for transforming sequences between binary format and function format (+1↔0 and -1↔1). These functions can be used for converting Walsh sequences to Walsh functions and for converting processed data.

Program contains several demonstrational functions. Two functions perform drawing of generated Walsh sequences and Walsh functions.

Next function draws complete procedure of spreading, despreading and reconstruction of transmitted data for code division purpose.

Other function demonstrates mergence and consequent division of two different data sequences. This function can show the collision, when two Walsh sequences from one subset are used for data coding.

Last function shows simple system with three channels. This function draw courses of three data samples during transmission by simplified wireless communication system. Figure generated by this function shows main principle of code division multiplex (Fig. 2).

User interface of this program is formed by simple window with four panels. There can be controlled all functions of program. Each panel contains text boxes for setting of function parameters. The length of processed data can be from 1 to 8 bits and order of Walsh function can be from 2 to 128 as mentioned above.

Program is completely created in Matlab.

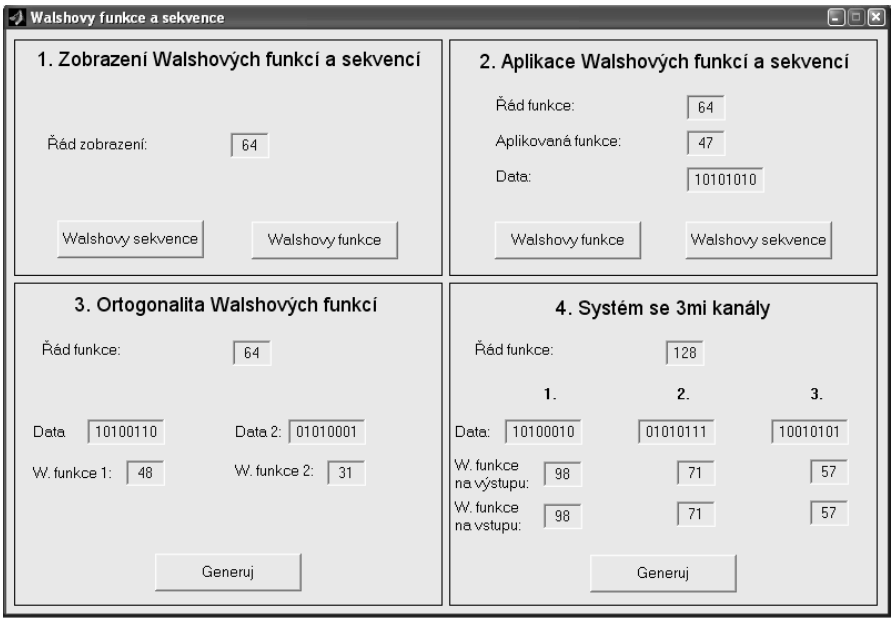

Fig. 1: *Application Window of Tutorial Program*

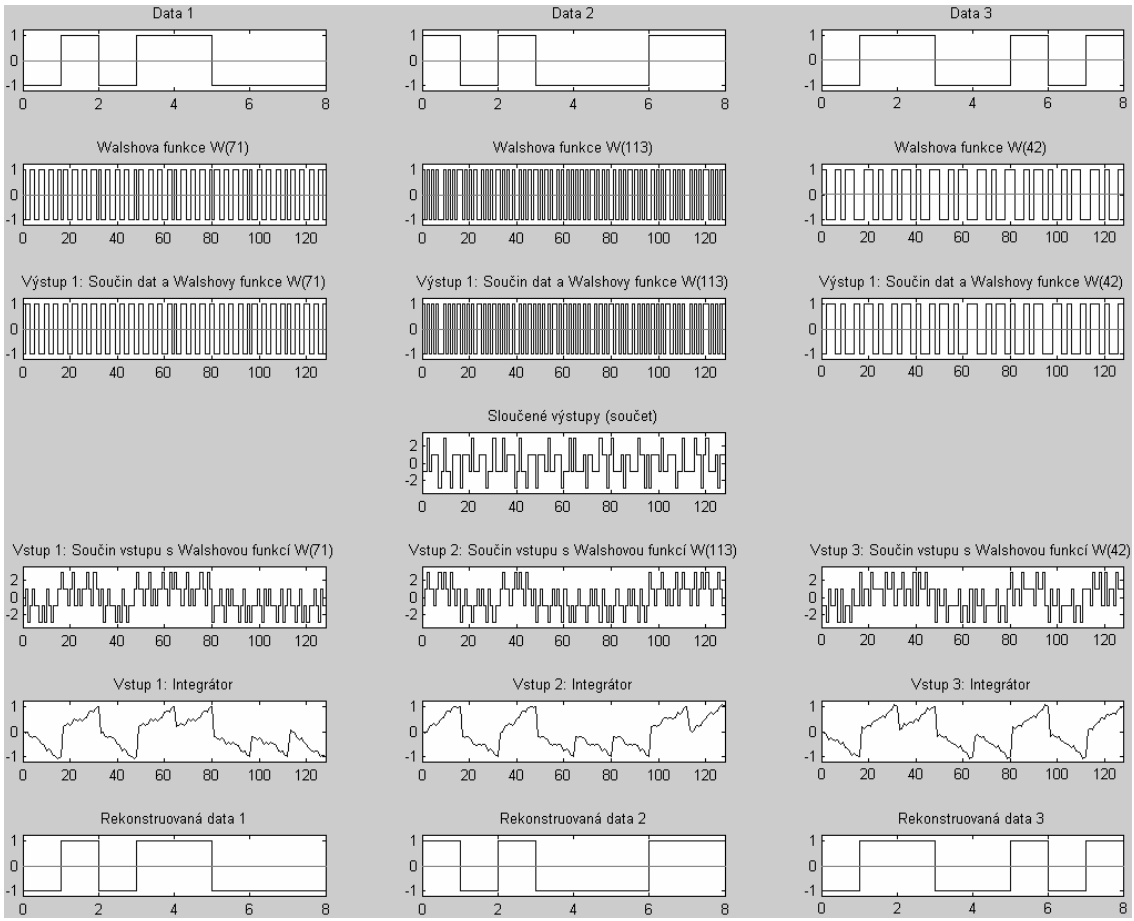

Fig. 2: *Demonstration of Code Division Multiplex Principle*

4 CONCLUSION

The article describes the process of Walsh functions a sequences generation using Rademacher functions. The article also introduces tutorial program which shows basic principles of code division multiplex as a fundamental part of CDMA and 3G wireless communication.

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REFERENCES

[1] Růžička, Z.: Elektrorevue: Walshovy funkce a způsoby jejich generování.[online]. Available: http://www.elektrorevue.cz/clanky/04065/, 2004

[2] Lee, J. S., Miller, L. E.: CDMA System Engineering Handbook. London: Artech House, 1998

[3] MATLAB Help: Creating Graphical User Interfaces