MODELLING OF ANTENNAS IN TIME DOMAIN USING LAGUERRE POLYNOMIALS

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ABSTRACT

The time-domain electric field integral equations (TD-EFIE) are solved by the method of moments. RWG functions are used as spatial basis and testing functions. The causal weighted Laguerre polynomials are used as temporal basis and testing functions. The approach for more accurate computing of transient responses in the time domain is introduced.

1 INTRODUCTION

Numerical techniques for the prediction of electro-magnetic fields scattered by complex objects, directly computing in time domain, have recently received considerable attention. Essentially, when broad-band information is desired, it is more efficient to solve the electromagnetic (EM) radiation or scattering problem in the time domain (TD). Several formulations have been presented for TD-EFIE [1], [2]. The marching-on in time method (MOT) and the implicit method have taken the most attention.

The MOT method suffers from late-time instability, which usually takes the form of an exponentially increasing oscillation that alternates in sign at each time step. In [1], the implicit method is proposed to remove the instability. This one is better, with reference to late-time oscillation, but not perfect [2]. In addition, the implicit method is less accurate than the MOT one. In [3], the unconditionally stable solution, with Laguerre polynomials, is proposed. This scheme does not suffer from the late time instability, but it is less efficient. There are five characteristic properties of the weighted Laguerre polynomials. They are causal, recursively computed, orthogonal, convergent and they separate the space and time variables.

This paper is focused on computing transient responses of planar antennas using weighted Laguerre polynomials as temporal basis and testing functions. The main attention is concentrated on the accuracy of solution and how it can be improved.

2 TIME-DOMAIN ELECTRIC FIELD INTEGRAL EQUATION FORMULATION

Let S denote the surface of a closed or open perfect electric conducting (PEC) body illuminated by a transient electromagnetic pulse. By enforcing the tangential electric field boundary condition on the PEC surface, the following integro-differential equation may be derived [1]

$$\left[\frac{\partial^2 \mathbf{A}(\mathbf{r},t)}{\partial t^2} + \nabla \psi(\mathbf{r},t)\right]_{\tan} = \left[\frac{\partial \mathbf{E}^i(\mathbf{r},t)}{\partial t}\right]_{\tan}, \quad \mathbf{r} \in S. \tag{1}$$

The vector potential $\mathbf{A}$ and the time derivative of the scalar potential ψ contain the unknown current density $\mathbf{J}(\mathbf{r}, t)$, which is induced by the incident electric field $\mathbf{E}^i$, $\mathbf{r}$ is an arbitrarily located observation point. The equation (1) is solved by the method of moments (MoM) [1]. The body is modeled with triangular patches. The unknown current density $\mathbf{J}(\mathbf{r}, t)$ is approximated as

$$\mathbf{J}(\mathbf{r},t) = \sum_{n=1}^{N} I_n(t)\mathbf{f}_n(\mathbf{r}), \tag{2}$$

where

$$\mathbf{f}_n = \begin{cases} \dfrac{l_n}{2A_n^{\pm}}\rho_n^{\pm}, & \mathbf{r} \in T_n^{\pm} \\ \mathbf{0}, & otherwise \end{cases} \tag{3}$$

In (2), (3) $I_n(t)$ is the unknown coefficient representing the value of the component of the surface current normal to the nth edge, f_n is the basis function, l_n is the length of the edge, $A_n^{\pm}$ is the area of the triangle $T_n^{\pm}$, $\rho_n^{\pm}$ is the vector from resp. to the free vertex of $T_n^{\pm}$ and N is the number of nonboundary edges. A boundary edge is defined as an edge, which is associated with only one triangular patch. These basis functions are called RWG functions and their properties are described in [1].

2.1 TD-EFIE WITH WEIGHTED LAGUERRE POLYNOMIALS

The temporal coefficients in (2) can be expanded as [4]

$$I_n(t) = \sum_{u=0}^{\infty} I_{n,u}\varphi_u(\bar{t}), \tag{4}$$

where $\varphi_u(\bar{t}) = e^{-s\frac{t}{c}} L_u(st)$ are weighted Laguarre polynomials $L_u(st)$ of order u, $\bar{t} = ts$ is the scale time. After the testing procedures, in the space and time, it can be written

$$\sum_{n=1}^{N}\sum_{p=1}^{2}\sum_{q=1}^{2}\left(s^2 a_{mn}^{pq} + b_{mn}^{pq}\right) = V_m^v, \tag{5}$$

$$m = 1,\ 2,\ 3\ldots\ N$$

where

$$a_{mn}^{pq} = \frac{\mu_0 l_m l_n}{16\pi}\rho_m^p(\mathbf{r}_m^c).\frac{1}{A_n^q}\int_{T_n^q}\frac{\rho_m^q(\mathbf{r}')aI_n(R_{mn}^p)}{R_{mn}^p}ds', \tag{6}$$

$$b_{mn}^{pq} = \frac{kl_m l_n}{4\pi\varepsilon_0}\frac{1}{A_n^q}\int_{T_n^q}\frac{bI(R_{mn}^p)}{R_{mn}^p}ds', \tag{7}$$

$$aI_n(R_{mn}^p) = \sum_{u=0}^{v} \left(0.25 I_{n,u} + \sum_{j=0}^{u-1} (u-j) I_{n,j} \right) \varphi_{v,u} \left(\frac{sR_{mn}^p}{c} \right), \tag{8}$$

$$bI_n(R_{mn}^p) = \sum_{u=0}^{v} I_{n,u} \varphi_{v,u} \left(\frac{sR_{mn}^p}{c} \right), \tag{9}$$

$$V_m^v = \frac{l_m}{2} \sum_{p=1}^{2} \rho_m^p(\mathbf{r}_m^c) . \int_0^\infty \frac{\partial \mathbf{E}^i(\mathbf{r}, t)}{\partial t} \varphi_v(\bar{t}) d\bar{t} \ , \tag{10}$$

$$\varphi_{v,u} \left(\frac{sR_{mn}^p}{c} \right) = \varphi_v \left(\frac{sR_{mn}^p}{c} \right) - \varphi_u \left(\frac{sR_{mn}^p}{c} \right), \tag{11}$$

$$R_{mn}^p = \left| \mathbf{r}_m^c - \mathbf{r'}_n \right|, \tag{12}$$

$$for\ v = 0, 1,\ 2, \ldots, M \ .$$

where p or $q=1$ means that p or q is "+" and p or $q=2$ means that p or q is "+", if $p+q$ is even number then $k=+1$, otherwise $k=-1$. v denotes the order of temporal testing function. $\mathbf{r}_m^c$ is the position vector of the centroid in triangle $T_m^\pm$.

It is needed the finite number M of temporal basis functions. This parameter is dependent on the time-bandwidth product of the excitation signal. It is considered a signal with bandwidth B in the frequency domain and of duration T_f in the time domain. M should satisfy this condition [3]

$$M \geq 2BT_f + 1 \,. \tag{13}$$

The upper limit of the integral in (10) can be replaced by a time duration sT_f instead of infinity.

The way of computing of the integrals in (6), (7) has the influence on the accuracy. There are several approaches. In [3], the distance R_{mn}^p is replaced, in (8) and (9), by $R_{mn}^{pq} = \left| \mathbf{r}_m^c - \mathbf{r}_n^c \right|$, then the functions aI_n and bI_n are collected in front of the integrals. After that, the integrals can be computed analytically. This way of computing is the fastest, but less accurate. In [4], the integrals are computed numerically. This approach is accurate, but very slow. The best solution could be chosen as a compromise between these ways. The suitable approach is approximating of the functions aI_n and bI_n on triangular patches by second-order polynomial.

2.2 APPROXIMATING FUNCTIONS aI_n AND bI_n ON TRIANGULAR PATCHES $T_n^\pm$ BY SECOND-ORDER POLYNOMIAL

The functions $aI_n(R_{mn}^p)$ (8) and $bI_n(R_{mn}^p)$ (9) can be evaluated on triangle $T_n^\pm$. For approximation by second-order polynomial, a common scheme could be used [5]. For this one, it is necessary to evaluate $aI_n(R_{mn}^p)$ and $bI_n(R_{mn}^p)$ for 3 distances, $R_{mn}^{p\min}, R_{mn}^{pcen}, R_{mn}^{p\max}$ (fig. 1). The distance $R_{mn}^{p\min}$ resp. $R_{mn}^{p\max}$ is the minimum resp. maximum distance between $\mathbf{r}_m^c$ on T_m^q and the nearest resp. furthest point on T_n^q. Further, R_{mn}^{pcen} is defined as $R_{mn}^{pcen} = (R_{mn}^{p\min} + R_{mn}^{p\max})/2$ and dR_{mn} is defined as $dR_{mn} = R_{mn}^{p\max} - R_{mn}^{pcen}$. The functions

$aI_n(R_{mn}^p)$ and $bI_n(R_{mn}^p)$ can be easily approached in interval $R_{mn}^p \in (R_{mn}^{p\min}, R_{mn}^{p\max})$

$$\begin{aligned} aI_n^a(R_{mn}^p) = \frac{1}{2dR_{mn}^{\;2}}\Big(&aI_n(R_{mn}^{p\min})((R_{mn}^p - R_{mn}^{pcen})^2 - (R_{mn}^p - R_{mn}^{pcen})dR_{mn} \\ &- aI_n(R_{mn}^{pcen})((R_{mn}^p - R_{mn}^{pcen})^2 - dR_{mn}^{\;2}) \\ &+ aI_n(R_{mn}^{p\max})((R_{mn}^p - R_{mn}^{pcen})^2 + (R_{mn}^p - R_{mn}^{pcen})dR_{mn})\Big) \end{aligned}, \tag{14a}$$

$$\begin{aligned} bI_n^a(R_{mn}^p) = \frac{1}{2dR_{mn}^{\;2}}\Big(&bI_n(R_{mn}^{p\min})((R_{mn}^p - R_{mn}^{pcen})^2 - (R_{mn}^p - R_{mn}^{pcen})dR_{mn} \\ &- bI_n(R_{mn}^{pcen})((R_{mn}^p - R_{mn}^{pcen})^2 - dR_{mn}^{\;2}) \\ &+ bI_n(R_{mn}^{p\max})((R_{mn}^p - R_{mn}^{pcen})^2 + (R_{mn}^p - R_{mn}^{pcen})dR_{mn})\Big) \end{aligned}, \tag{14b}$$

The meaning of symbols is evident in the fig. 1.

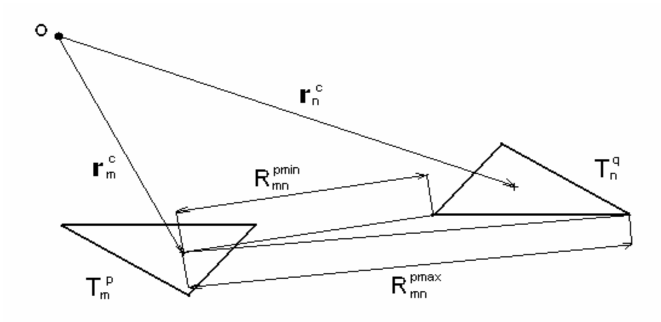

Fig. 1: *The definition of the distances* $R_{mn}^{p\min}, R_{mn}^{p\max}$

After approximation, the function $aI_n(R_{mn}^p)$ resp. $bI_n(R_{mn}^p)$ can be replaced by $aI_n^a(R_{mn}^p)$ in (6) resp. by $bI_n^a(R_{mn}^p)$ in (7). Now, both integrals can be easily computed analytically.

The approximation by second-order polynomial is chosen, because $aI_n(R_{mn}^p)$ and $bI_n(R_{mn}^p)$ are piecewise quadratic functions as computing current transient responses. According to our experience, for obtaining of satisfied results, the following condition must be satisfy

$$2dR \leq \frac{c}{10B}, \tag{15}$$

where dR is the minimum value of all dR_{mn} and c is the velocity of wave in the space.

3 NUMERICAL EXAMPLE

The presented approximation was verified on computing the current in the center of the strip, which was excited as an antenna by a Gaussian pulse [1]. The strip, 2.00 m x 0.01m, is located along X-axis. It is divided in the x direction into 20 rectangular patches. By joining diagonals of each rectangle, we have 40 triangular patches with 39 unknowns.

This strip was solved for: M=250, B=250 MHz. The results are in the fig. 2. The comparison is made with frequency domain data that were inversely Fourier transformed for fig. 2a). The difference isn't seen in the fig. 2a). However, in the fig. 2b) where time domain data were Fourier transformed, the difference is seen, but above the desired frequency B.

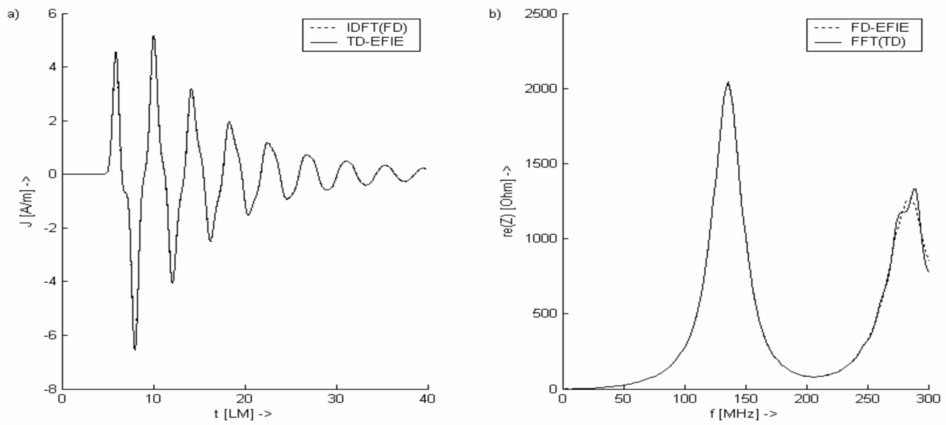

Fig. 2: *The transient current response a) and the real part of input impedance b) at the center of the strip*

4 CONCLUSION

In this paper, the approach for more accurate computing of transient responses in the time domain has been introduced. It was verified on computing the current in the center of the strip, which was excited as an antenna. The comparison was made with frequency domain data in the time and frequency domain. The results are very accurate until the desired frequency.

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