SIMULATION AND COMPUTATION OF OFDM AND PEAK REDUCTION CARRIERS SCHEME

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ABSTRACT

One of the main disadvantages of Orthogonal frequency division multiplex (OFDM) is high peak-to-average power ratio (PAPR). In this article, the effect of oversampling to compute PAPR and its CDF (Cumulative Distribution Function) is examined. Next, CDF for one example of application of PRC (Peak Reduction Carriers) is analyzed.

1 INTRODUCTION

Orthogonal frequency division multiplex (OFDM) is mainly used for wireless communication systems. The basic idea of OFDM is to split high-speed data to low-speed parallel channels. Longer symbol duration is the way to combat with multipath fading. The cyclic prefix is implemented to enhance this. The effects of selective fading can be suppressed by applying FEC coding on source data. N parallel orthogonal channels are obtained by using IFFT. The FFT is used for demodulation.

The main drawback of OFDM is its high PAPR. Signal with large peaks can be obtained by constructive superposition of subcarriers. PAPR depends linearly on number of subcarriers, but in the systems with large number of subcarriers, probability of symbol with large PAPR is smaller. This leads to use CDF (Cumulative Distribution Function) to describe PAPR distribution. High peak power means disadvantages with power amplifiers, A/D and D/A converters. Due to amplifier imperfection, peaks are distorted nonlinearly. The intermodulation product occurs as effect of nonlinear distortion. They can be interpreted as inter carrier interferences (ICI) and out-of-band radiation.

In the simulations, PAPR is evaluated from the discrete time samples. The sampling rate is at least Nyquist or greater (oversampling). In this work the effect of oversampling on calculation of PAPR is mentioned. In the next part, PRC (Peak Reduction Carriers) reduction scheme is simulated. PRC is a reduction scheme, where new subcarriers are inserted to obtain symbol with lower PAPR.

2 OFDM SYSTEM MODEL

2.1 MODULATION

The OFDM symbol consists of N subcarriers, which have constant spacing Δf. The bandwidth of that signal is $B = \Delta f \cdot N$ and symbol time $T = 1/\Delta f$. In time domain, this leads to sum of N sinusoids, which have exactly an integer number of cycles in the interval T. Each subcarrier is modulated by complex value $X_{m,n}$, where m denotes symbol index and n subcarrier index.

M-th OFDM symbol can be written as:

$$x_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT), \qquad (1)$$

where $g_n(t)$ means: $g_n(t) = \exp(j2\pi n \Delta f t)$, for $0 \le t \le T$ and

$g_n(t) = 0$, for other t.

Time domain signal can be written as a sum of symbols:

$$x = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT). \qquad (2)$$

The complex value $X_{m,n}$ depends on subcarrier modulation. Usually M-PSK or M-QAM is used.

2.2 PEAK POWER PROBLEM

OFDM consists of many independent modulated subcarriers (without considering coding). This leads to problem with peak to average power ratio. If N subcarriers are in phase (same symbols on all subcarriers), the peak power is N times average power. For sampled signal, PAPR is defined:

$$PAPR = \frac{\max |x_m|^2}{\mathrm{E}\left[|x_m|^2\right]}, \qquad (3)$$

where $\mathrm{E}\left[|x_m|^2\right]$ is average power of transmitted symbol. Oversampling can be performed by padding IFFT source data with zeros.

2.3 PAPR DISTRIBUTION

PAPR can take values in range that is proportional to the number of subcarriers. Probability of symbol with maximal PAPR depends on number of subcarriers. Therefore, symbols with high PAPR have small probability in the systems with large number of subcarriers. Cumulative distribution function is used to formulation of PAPR probability.

Cumulative distribution function is the probability that the variable *X* takes a value less than or equal to x:

$$F(x) = \Pr[X \leq x]. \qquad (4)$$

For a discrete distribution, the CDF can be expressed as:

$$F(x) = \int_{-\infty}^{x} f(x)dx \text{, where } f(x) = \sum_{k=1}^{K} \delta(x - x_k)P(x_k) \qquad (5)$$

and $P(x_k) = \Pr[X = x_k]$, where x_k represents possible values of X.

2.4 PEAK REDUCTION CARRIERS

PRC (Peak Reduction Carriers) can be inserted to the OFDM symbol to achieve low PAPR. Phases and amplitude of these carriers are set to minimalise PAPR. These carriers are redundant, but can be used e.g. for error detection. The original information carriers are unaffected and PRC can be inserted without decoding of information on original carriers.

The optimum setting of PRC depends on phase and amplitude of carrier and position of carrier in the OFDM symbol.

3 SIMULATIONS

The OFDM modulation with 12 subcarriers, each modulated with BPSK was made in Matlab. All possible symbols (2^{12}) ware calculated and CDF was plotted. This was made for Nyquist sampling (L=1) and for oversampling L-times (fig. 1).

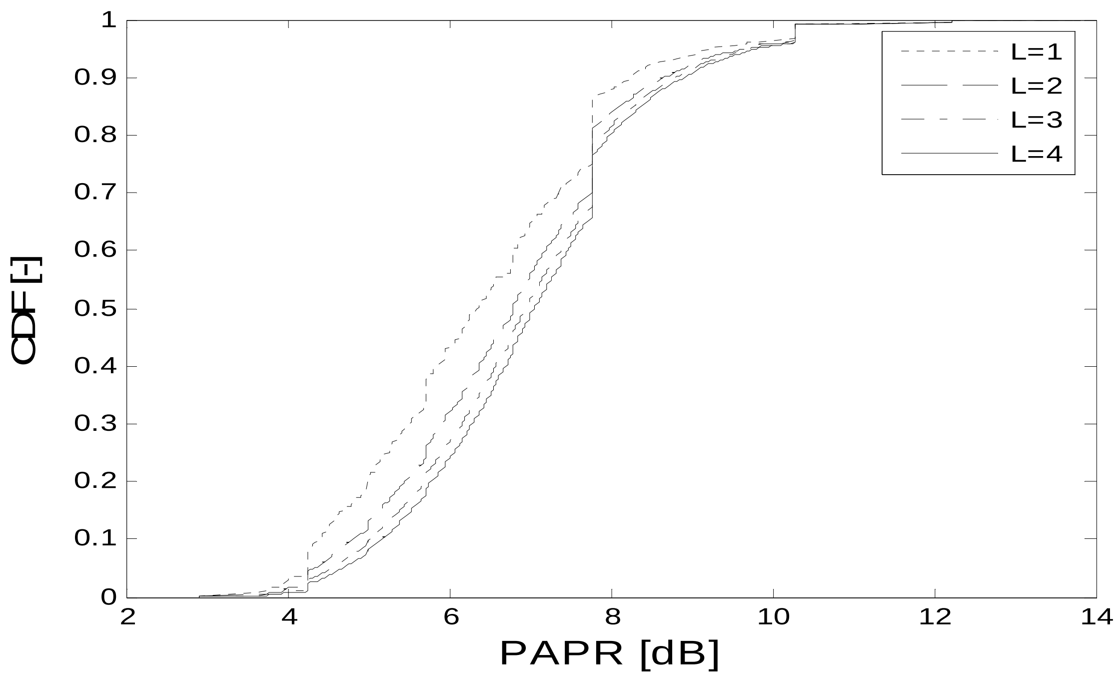

Fig. 1: *Cumulative distribution function of PAPR for OFDM (12 subcarriers, BPSK modulated)*

It can be seen that CDF is closing to limit (right value) with increasing oversampling factor L. Up to 10 % difference in CDF may occurs without oversampling. Oversampling factor of four is usually used for simulations.

This oversampling is used in next simulation. CDF is evaluated for same OFDM data, but now PRC is used. Ten information carriers and two peak reduction carriers are used in the symbol. CDF of symbols without PRC can be seen on fig. 1.

From fig. 2 can be seen that CDF of OFDM with PRC depends on position of PRC. The peak reduction carriers effects not only maximum of PAPR, but also distribution of PAPR. Symbols with PAPR < 4dB can bee seen in OFDM symbol without PRC (fig.1), but not in symbol with PRC (fig. 2). It is because of same amplitude and phase of PRC. Better results can be obtained by independent PRC at the cost of computing complexity.

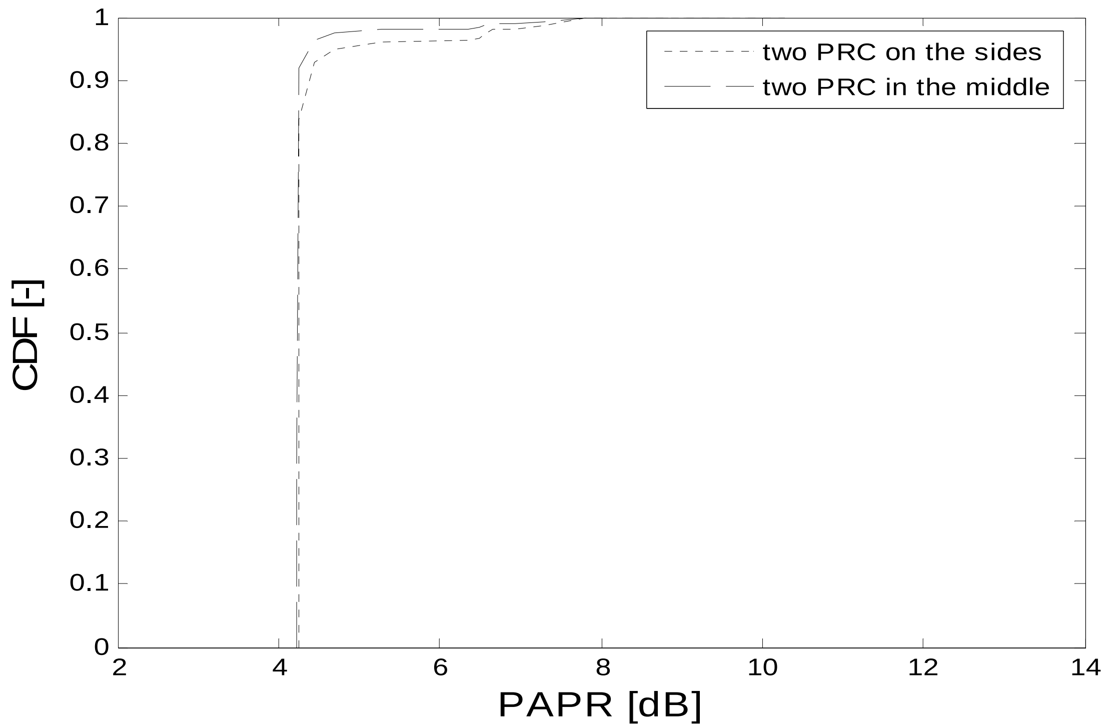

Fig. 2: *Cumulative distribution function of PAPR for OFDM with peak reduction carriers (12 subcarriers, BPSK modulated, 2 PRC)*

4 CONCLUSION

The demand of oversampling in calculations of CDF for PAPR in OFDM symbols is examined in this article. Four-time oversampled signal provides sufficient accuracy in calculation of CDF. This is used in examination of PRC via CDF of PAPR. CDF allows keep under review whole set of possible OFDM symbols. The plot of CDF shows differences in PRC when same number of PRC is used but their position is changed.

ACKNOWLEDGEMENTS

The paper has been prepared as a part of the solution of GAČR projects No. 102/03/H109 and 102/04/0557.

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