VISUALIZATION OF 3D POTENTIAL FIELDS COMPUTED USING TKSL/C

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ABSTRACT

The aim of this paper is to examine the possibility of visualization of potential fields. The numerical data for the visualization process are computed by simulation system TKSL/C. 3D potential fields are difficult to present; that is why the Particle Tracing method is harnessed.

1 INTRODUCTION

Partial differential equation (PDE) is an equation involving functions and their partial derivatives. PDEs are much more difficult to solve than ordinary differential equations and very often they have to be solved by a numerical method.

The Modern Taylor Series Method (MSMT) provides an extreme fast and accurate way to solve a system of differential equations. The TKSL/C is a simulation program that utilizes MSMT. It is currently being developed by the High Performance Computing Research Group at the Department of Intelligent Systems, FIT BUT. Having these starting conditions it was the aim of the work to adapt TKSL/C for solving PDE.

The method of Particle Tracing enables human user of any process of calculation or generation of 3D vector or scalar (potential) field to visualize the achieved results. Visualization of 1D and 2D fields is quite a simple task, while visualization of 3D fields using

2D display leads to significant loss of information. One effective solution is to make the visualization temporal: insert a number of particles into the field, let the field influence their kinematics and visualize the motion of the particles.

2 ELLIPTIC PDE

Let us examine an elliptic partial differential equation (Laplace's equation)

$$\nabla^2 u = 0 \tag{1}$$

in 3D with some initial boundary conditions. The equation has to be transformed to a form suitable for TKSL/C (i.e. to a system of ordinary differential equations).

The equation (1) can be written as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{2}$$

Now we can replace the right side of the equation by $\frac{\partial u}{\partial t}$, and the second derivatives using an approximation, getting thus

$$\frac{u_{i-1} - 2u + u_{i+1}}{(\triangle x)^2} + \frac{u_{j-1} - 2u + u_{j+1}}{(\triangle y)^2} + \frac{u_{k-1} - 2u + u_{k+1}}{(\triangle z)^2} = \frac{\partial u}{\partial t} \tag{3}$$

Supposing that the examined area is divided with the step h in all directions we get an ODE system

$$\vdots$$
$$u'_{i,j,k} = \frac{1}{h^2}(u_{i-1,j,k} + u_{i+1,j,k} + u_{i,j-1,k} + u_{i,j+1,k} + u_{i,j,k-1} + u_{i,j,k+1} - 6u_{i,j,k}) \tag{4}$$
$$\vdots$$

If we find the steady state of system (4) we will get solutions of the $u(i,j,k)$.

3 VISUALIZATION BY PARTICLE TRACING

Let us consider a vector field of 3D vectors (it can be built out of a potential field just using the ∇ operator). We insert particles of zero size, mass m, initial position $\vec{r}_0$ and initial speed $\vec{v}_0$ (that may be $\vec{0}$). Every particle P_i is affected by the vector field $\vec{f}(\vec{r})$: $\vec{v}_i' = \frac{\vec{f}(\vec{r}_i)}{m_i}$ and every particle is moving through the examined region of the 3D space:

$$\vec{v}_i = \frac{\partial \vec{r}_i}{\partial t}. \tag{5}$$

If the particle leaves the examined region of space, it is re-inserted from the particle emitter. Simple Euler integration by time quantums in real time and concurrent visualization explores the 3D vector (or potential) field effectively.

Some degree of interactiveness improves the visualization significantly:

- The user can change the projection parameters: rotation, scale, etc.

- The user can define parameters of the particle emitter: its shape (rectangular surface or volume etc.), number of particles emitted within a time unit, initial speed $\vec{v}$ (defined as a vector range), etc.

- Sometimes it is useful to modify the weights of the particles m_i.

- The visualization may be configured by numerous other parameters in real-time.

There are several approaches to particles' coloring. Different scalar parameters of the particles can be visualized this way: speed, speed change (acceleration), direction change, measure of influencing force, etc. Either the brightness (intensity) of the particle is controlled, or one or more scalar parameters determine a color from a color gradient (Hue, ramp from one color to another, . . .).

Since it would be inadequate to compute too much equations and interpolation algorithm was used. Our research in the past [5] resulted in an interpolation algorithm operating on octree-based hierarchical structures, which offers adaptive sub-optimal interpolation on a structure with some degree of data compression.

4 RESULTS

The PDE (2) has been solved within a cubic area $\Omega : (x,y,z) \in < 0,0,0 > - < 1,1,1 >$ with the following initial boundary conditions: for the particle emitting side is $u(x,y) = 0$, for the opposite side $u(x,y) = 4\sin(\pi x)\sin(\pi y)$, all the other sides have $u = 1 - z$. An example of obtained results is shown in Fig. 1.

Some auxiliary programs have been created for generation of system in form (4) with the initial boundary condition and another for computation of vector fields needed by the particle tracing algorithm.

In this way it was possible to study the behavior of the TKSL/C when solving large systems of equations: TKSL/C successfully solved systems of tens thousands equations. During the tests was exploited a unique feature of MSMT – the automatic integration step setting which sped up the computation considerably.

5 DISCUSSION

- In order to get reasonable results we have to cover the examined area with a dense-enough grid of points where we actually compute the solutions. Unfortunately, this leads to very large systems of differential equations, especially in case of 3D problems.

- Making the grid denser introduces a new problem: the obtained system of ordinary differential equations may become *stiff* which is generally difficult to solve due to the short integration step that has to be used during the computation [3].

- Solving and visualizing a large system requires lots of computer memory and powerful processor.

The new simulation program TKSL/C which utilizes Modern Taylor Series Method, optionally the arbitrary long arithmetic (using the GMP library) and a new approach to solution of stiff systems deals with the issues:
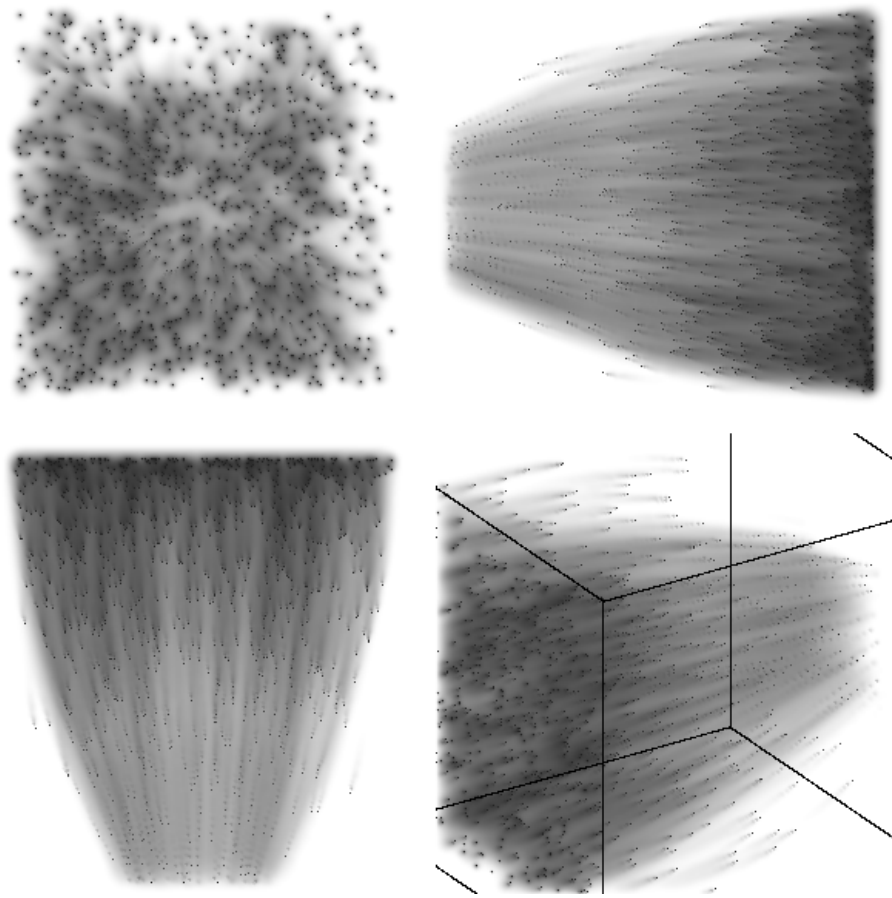
Figure 1: Potential field as computed by TKSL/C and displayed using the particle tracing method

- In order to deal with the first problem a new version of simulation language TKSL – TKSL/C has been created. The syntax of the input has been simplified, the number of equations is not limited, the process of computation can be automatized.

- A new method for solving stiff equations [3] is now being integrated with the new TKSL/C and is expected to reduce the amount of computation work considerably. Besides, TKSL/C is now capable of using arbitrary wide arithmetics which has a positive effect on the computation, too [2].

- Since the MSMT requires only the basic mathematical operations (+,-,*,/) for the calculations, very simple specialized elementary processors can be designed for their implementation thus creating an efficient parallel computing system with a relatively simple architecture [4].

The visualization algorithm can be improved in several ways:

- Some modifications, especially to the traces (tails) of the particles, could be done in order to increase the amount of human-perceptible information in static images.

- Particle tracing and rendering could be done in parallel, using massively parallel hardware architectures [6]. At the moment, hardware acceleration of particle tracing and rendering is one of our primary focuses.

- Future development and improvements will be presented within the Jim project [7] accessible on the web.

6 CONCLUSION

The simulation program TKSL/C is now usable for solving partial differential equations. Because of the unique features of TKSL/C the solution is obtained with both great accuracy and speed.

By using the Particle Tracing method it is possible to present the results in a very clear way. This way of visualization is in a big measure relying in real-time animation and interactiveness – static images produced by this algorithm are not as clear.

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REFERENCES

[1] Kunovský, J.: Modern Taylor Series Method, Habilitation work, FEI VUT Brno, 1995

[2] Kunovský, J., Řezáč, D., Petřek, J.: TKSL/C an Multiple Arithmetic, In: Proceedings of the Fifth International Scientific Conference Electronic Computers and Informatics 2002, Košice, SK, 2002, p. 19-21, ISBN 80-7099-879-2

[3] Řezáč, D.: Solving Stiff Systems in Parallel Utilizing the Taylor Series Method, In: Proceedings of 8th Conference STUDENT EEICT 2002, Brno, CZ, FEKT VUT, 2002, p. 486-490, ISBN 80-214-2116-9

[4] Kunovský, J., Václavík, P., Řezáč, D.: Parallel Computations Based on the Taylor Series Method, In: Proceedings of 4th MATHMOD Vienna, 2003, Vienna, AT, ISBN 3-901608-24-9

[5] Tišnovský, P., Herout, A.: Adaptive Algorithm for Vector Field Interpolation Based on Octree Structure, SCCG 2002, Budmerice, 2002

[6] Zemčík P.: Hardware Acceleration of Graphics and Imaging Algorithms Using FPGAs, SCCG 2002, Budmerice, 2002

[7] Tišnovský, P., Herout, A.: Jim - Physical Vector Field Visualization, Research Project, DCGM, Faculty of Information Technology, Brno University of Technology, 2002, (home page available at `http://www.fee.vutbr.cz/~tisnovpa/jim`)