ANALYSIS OF WIRE DIPOLE BY TIME DOMAIN MOMENTS METHOD

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ABSTRACT

This thesis deals with computing current distribution and input impedance of wire dipole by Moment method (MoM) in time domain. The wire antenna is fed by uniform Gaussian plane wave. With know input electrical intensity (or voltage), we can compute the output current distribution. Thus we can compute input impedance from input voltage and output current by Fast Fourier transformation (FFT) in all wideband area. This method is faster then computing impedance on singles frequencies.

1 INRODUCTION

Principe of this method is shown in **Fig. 1**. We excite the unknown system $H(Z)$ by known input $X(Z)$ (in our case, Gaussian pulse of electrical intensity or voltage), and then, we compute the response of the system $Y(Z)$; in our case, current is the response. Then, we can compute properties of the system by (1) (in our example impedance of system).

$$H(Z) = \frac{X(Z)}{Y(Z)} \qquad (1)$$

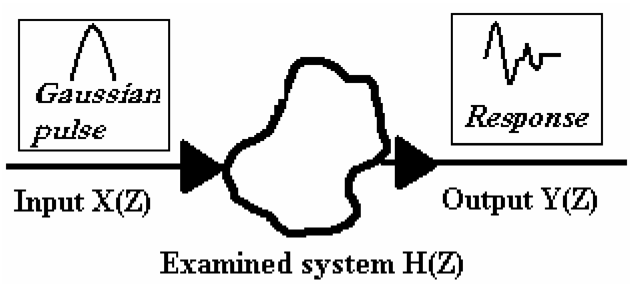

Fig. 1: *Principe of method*

1.1 GAUSSIN PULSE

Gaussian pulse is defined as

$$E^i(r,t) = E_0 \frac{4}{T\sqrt{\pi}} e^{-\gamma^2} \tag{2}$$

with

$$\gamma = \frac{4}{T}(ct - ct_0 - \bar{r} \cdot \bar{a}_k) \tag{3}$$

where a_k is unit propagation vector of incident wave, T is pulse width, c is free-space velocity of propagation, t_0 is time when pulse peak appears, and E_0 is magnitude of the intensity in the peak. Gaussian pulse is shown in **Fig. 2**.

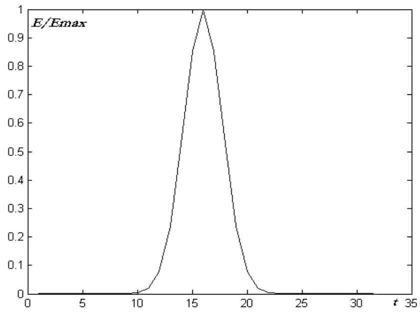

Fig. 2: *Gaussian pulse*

2 ANALYSIS OF A STRAIGHT WIRE

In this section, integral equations are derived for the current induced on a thin, finite-length perfect conducting cylindrical tube excited by an incident electromagnetic field. The incident electric field $E^i(r,t)$ is assumed to be invariant around the cylinder circumference and polarized along the length of the cylinder.

Consider a thin wire of length $2h$ and radius a located symmetrically along z-axis as shown in the **Fig. 3**. The incident electric field induces current $I(z, t)$, which is a function of z and t only because of the thin-wire approximation. Using the reduced kernel approximation, the vector potential is given by

$$A_z(x,y,z,t) = \mu \int_{z'=-h}^{h} \frac{I(z', t - R/c)}{4\pi R} dz', \tag{4}$$

where

$$R = \sqrt{[x^2 + y^2 + (z - z')^2 + a^2}, \tag{5}$$

and a is radius of the wire.

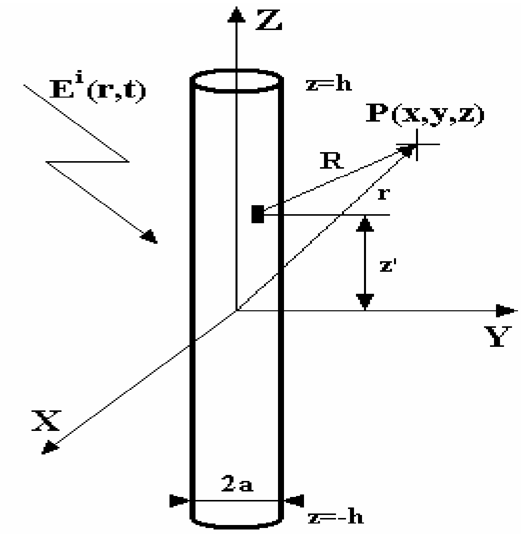

Fig. 3: *Finite length wire excited by an incident electromagnetic plane wave*

The total electric field is the sum of the incident and scattered fields. Next, we apply the boundary condition for total electric field, which implies that the *z* component of the total electric field must vanish on the conducting surface. Thus, we derive the electric field integral equation for a straight setting $x = y = 0$, in Eq. (5), and noting that $\nabla(\nabla.A) = (\partial^2 A_z)/(\partial z^2)$, given by

$$\frac{\partial^2 A_z}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_z}{\partial t^2} = -\frac{1}{c^2}\frac{\partial E_z^i}{\partial t} \text{ for } z \in (-h, h), \tag{6}$$

where E_z^i is the *z* component of incident electric field E^i, and A_z is given by

$$A_z(z,t) = \mu \int_{z'=-h}^{h} \frac{I(z', t - \frac{|z - z'|}{c})}{4\pi\sqrt{|z - z'|^2 + a^2}} dz'. \tag{7}$$

2.1 METHOD OF MOMENTS SOLUTION

First, a set of basic function for expansion purposes is defined [1], given by

$$f_m(z) \equiv \begin{cases} 1 & z_m - \frac{\Delta z}{2} \le z \le z_m + \frac{\Delta z}{2} \\ 0 & otherwise \end{cases} \tag{8}$$

These expansion functions are just the standard pulse functions on a linear segment. Using these expansion functions, we approximate the current *I* as

$$I \approx \sum_{k=1}^{N} I_k(t) f_k(z), \tag{9}$$

where $I_k(t)$ is unknown coefficient at the *k*th zone. Denoting $A_{m,n} = A_z(z_m, t_n)$[1], we may rewrite Eq. (7) as

$$A_{m,n} = \mu \int_{z'=-h}^{h} \frac{I(z', t_n - \frac{|z_m - z'|}{c})}{4\pi\sqrt{|z_m - z'|^2 + a^2}} dz' \approx \mu \int_{z'=-h}^{h} \frac{\sum_{k=1}^{N} I_k(t_n - \frac{|z_m - z'|}{c}) f_k(z')}{\sqrt{|z_m - z'|^2 + a^2}} dz' \tag{10}$$

$$= \sum_{k=1}^{N} I_k(t_n - \frac{|z_m - z'|}{c}) \kappa_{m,k}$$

where

$$\kappa_{m,k} = \mu \int_{z'=z_k-\Delta z/2}^{z_k+\Delta z/2} \frac{dz'}{4\pi\sqrt{|z_m - z'|^2 + a^2}} = \frac{\mu}{4\pi} \left\{ \begin{array}{l} \ln\left[z_m - z_k + \frac{\Delta z}{2} \sqrt{\left(z_m - z_k + \frac{\Delta z}{2} \right)^2 + a^2} \right] - \\ \ln\left[z_m - z_k - \frac{\Delta z}{2} \sqrt{\left(z_m - z_k - \frac{\Delta z}{2} \right)^2 + a^2} \right] \end{array} \right\}. \tag{11}$$

Eq. (11) is known as Green function for time domain.

Here we observe that we can also write Eq. (10) as

$$A_{m,n} = I_{m,n} \kappa_{m,m} + Ad_{m,n} \tag{12}$$

where

$$Ad_{m,n} = \sum_{\substack{k=1 \\ k \neq m}}^{N} I_k \left(t_n - \frac{|z_m - z_k|}{c} \right) \kappa_{m,n} . \tag{13}$$

Note that $Ad_{m,n}$ explicitly denotes that the $k=m$ term is omitted from the summation. We also observe that the current I_k in Eq. (10) is computed at time instants earlier than t_n because $|z_m - z_k| \neq 0$. This fact can be exploited to our advantage as follows: By selecting $c\Delta t \leq \Delta z$ we note that, even if $|z_m - z_k| = \Delta z$, the current I_k in the summation of Eq. (10) is known since it occurs at time instant $t \leq t_{n-1}$. Here, we are implicitly assuming that when the currents at time instant $t = t_n$ are computed, currents at earlier time instants $t = t_j$, $j = 1,\ldots,n-1$, are known. Thus, the only unknown in Eq. (12) is the term $I_{m,n}$.

3 RESULTS

In this section we examined with MoM the wire dipole of these attributes: length of dipole l= 2m, radius a=0.01m shown in **Fig. 3**. The Gaussian input plane wave pulse has these attributes: E_0=120π, ct_0=3 LM, T=2 LM, where LM denotes light meter. The dipole is divided into 100 identical sub-domains. Time step is restricted as 0.02 LM. Time response is 100 periods. The **Fig. 4** shows the time response of the current on the source slot (cell 50). The **Fig. 5** shows the current distribution on the dipole at the 50^{th} period. In the **Fig. 6**, we show the magnitude of the input impedance of the wire dipole computed by FFT. Theoretical values of resonance are 75MHz for quarter resonance and 150MHz for half resonance of this wire dipole.

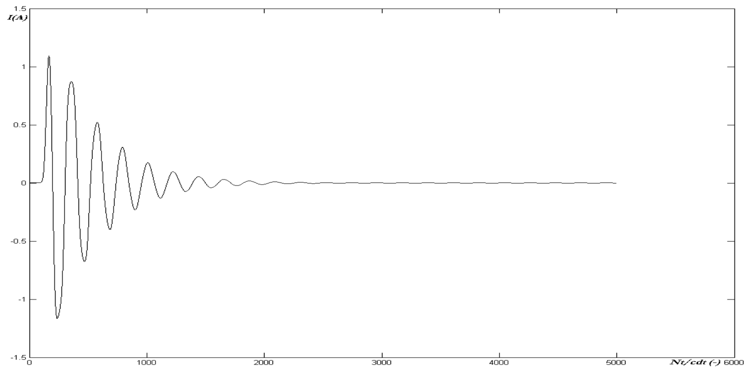

Fig. 4: *Current time response on the slot cell*

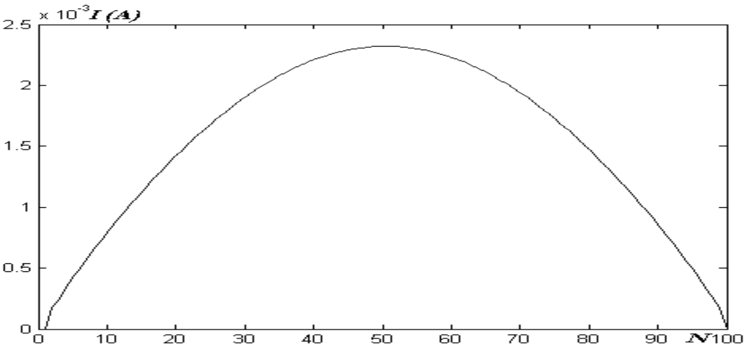

Fig. 5: *Current distribution of the wire dipole at the 50^{th} period*

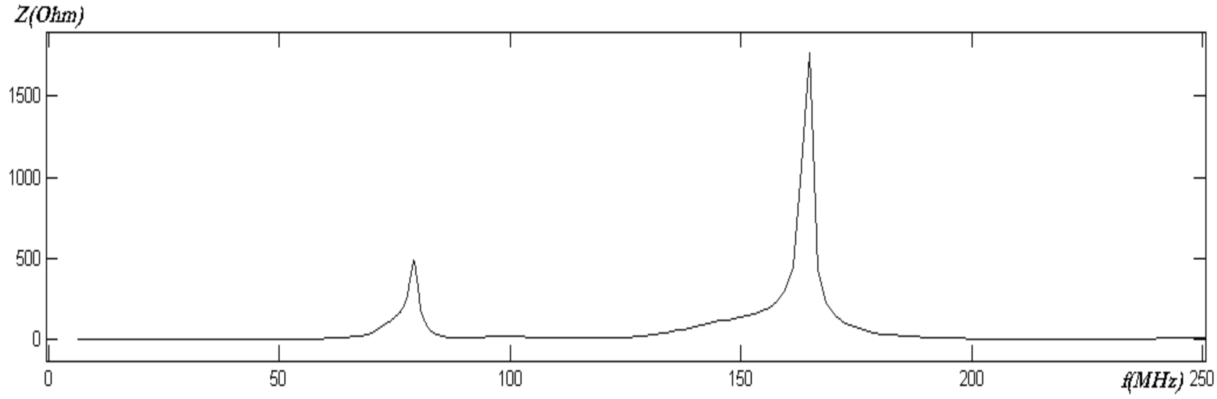

Fig. 6: *Magnitude of input impedance of the wire dipole*

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REFERENCES

[1] RAO, S. M. Time Domain Electromagnetics. Academic Press, p. 49-55, 1999